Discrete distributions

(sample space is a subset of the integers)

Discrete uniform(a, b) $a, b \in Z, a < b$

Equally likely events labelled with integers from a to b.

| PMF | 1/n n = b - a + 1 |
|----------|---------------------------|
| Support | $\{a, a + 1,, b - 1, b\}$ |
| Mean | $\frac{a+b}{2}$ |
| Variance | $\frac{n^2 - 1}{12}$ |

Bernoulli(p) p ∈ [0, 1]

An event with two outcomes: 1 = success, 0 = failure

| PMF | 1 - p x = 0 |
|----------|----------------|
| | p x = 1 |
| Support | $\{0,1\}$ |
| MGF | $1 - p + pe^t$ |
| Mean | p |
| Variance | p(1-p) |

Binomial(n, p) n = 1, 2, 3, ...; $p \in [0, 1]$

The number of successes in n Bernoulli(p) trials (a count)

| PMF | $\binom{n}{x} p^x (1-p)^{n-x}$ |
|----------|--------------------------------|
| Support | $\{0, 1,, n\}$ |
| MGF | $(1 - p + pe^t)^n$ |
| Mean | np |
| Variance | np(1-p) |

$\textbf{Geometric(p)} \ p \in [0, 1]$

The number of successes in Bernoulli(p) trials until one failure.

| PMF | $(1-p)p^x$ |
|----------|----------------------|
| Support | $\{0, 1, 2,\}$ |
| MGF | $\frac{1-p}{1-pe^t}$ |
| Mean | $\frac{p}{1-p}$ |
| Variance | $\frac{p}{(1-p)^2}$ |

Negative binomial(r, p) $r = 1, 2, 3, ... p \in [0, 1]$ The number of successes in Bernoulli(p) trials until r failures.

| PMF | $\binom{x+r-1}{x}(1-p)^r p^x$ |
|----------|-------------------------------------|
| Support | $\{0, 1, 2,\}$ |
| MGF | $\left(\frac{1-p}{1-pe^t}\right)^r$ |
| Mean | $rrac{p}{1-p}$ |
| Variance | $r\frac{p}{(1-p)^2}$ |

Poisson(\lambda) $\lambda > 0$

Count of events that have constant rate, and are independent of one another.

| PMF | $\frac{\lambda^x e^{-\lambda}}{x!}$ |
|----------|-------------------------------------|
| Support | $\{0, 1, 2,\}$ |
| MGF | $\exp(\lambda(e^t - 1))$ |
| Mean | λ |
| Variance | λ |

Continuous distributions

(sample space is an interval on the real line)

Uniform(a, b) $a, b \in R$

Likelihood of event proportional to it's length.

| PDF | $\frac{1}{b-a}$ |
|----------|----------------------|
| CDF | $\frac{x-a}{b-a}$ |
| Support | [a,b] |
| Mean | $\frac{a+b}{2}$ |
| Variance | $\frac{(b-a)^2}{12}$ |

Exponential(\theta) $\theta > 0$

Waiting time until a Poisson event with average waiting time θ .

| PDF | $rac{1}{	heta}e^{-x/	heta}$ |
|----------|------------------------------|
| CDF | $1 - e^{-x/\theta}$ |
| Support | $[0,\infty)$ |
| MGF | $\frac{1}{1-\theta t}$ |
| Mean | θ |
| Variance | θ^2 |

Exponential(λ) $\lambda > 0$

Waiting time until a Poisson event with rate λ .

| PDF | $\lambda e^{-x\lambda}$ |
|------|-------------------------|
| Mean | $1/\lambda$ |

Gamma(a, \theta) $\alpha > 0, \theta > 0$ Waiting time for α Poisson events with average waiting time θ .

| PDF | $\frac{\theta^{-\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-x/\theta}$ |
|----------|--|
| CDF | No closed form |
| Support | $[0,\infty)$ |
| MGF | $\frac{1}{(1-\theta t)^{\alpha}}$ |
| Mean | $\alpha \theta$ |
| Variance | $\alpha \theta^2$ |

Gamma(\alpha, \beta) $\beta > 0, \theta > 0$

Waiting time for a Poisson events with rate β .

| PDF | $\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$ |
|------|---|
| Mean | lpha/eta |

Normal(\mu,\sigma^2) $\mu \in R$, $\sigma^2 > 0$

| PDF | $\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ |
|----------|---|
| CDF | $\Phi(x)$ |
| Support | $(-\infty,\infty)$ |
| MGF | $\exp(\mu t + \frac{1}{2}\sigma^2 t^2)$ |
| Mean | μ |
| Variance | σ^2 |