Homework 1 Model Answers Statistics 310

1. License plates in Texas (8 pts)

a. (2 pts) How many possible plates are there given that all license plates have the form letter - letter - number - letter - number - number - number

ANSWER: There are 26 possible letters A-Z and 10 possible numbers 0-9
So the total number of license plate combinations is

(1 point for A-Z = 26, 0-9 = 10)

$= 26 \times 26 \times 10 \times 26 \times 10 \times 10 \times 10$ (1 point for correct set up)

$= 175760000$ (176,000,000)

b. (2 pts) How many passenger plates are available, given that AA0-A001 to AZ9-Z999 and XA0-A001 to XZ0-Z999 are reserved for other types of plates?

ANSWER: Students should calculate the number of reserved plates and subtract it from the answer to 1.a.

AA0-A001 to AZ9-Z999 $= 1 \times 26 \times 10 \times 26 \times 10 \times 10 \times 10 - 1$ } ½ point

$= 6759999$ (6,760,000).

(we subtract 1 because AA0-A000 is apparently allowed. Do not take off points if students miss this)

XA0-A001 to XZ0-Z999 $= \underbrace{1 \times 25 \times 10 \times 26 \times 10 \times 10 \times 10}_{\text{XA0-A000 to XY9-Z999}} - \underbrace{1}_{\text{XA0-A000}} + \underbrace{1 \times 1 \times 1 \times 26 \times 10 \times 10 \times 10}_{\text{XZ0-A000 to XZ0-Z999}}$ } ½ point

$= 6525999$ (6,530,000)

(again do not take off points if students forget to subtract 1)

Remaining passenger plates $= 175760000 - 6759999 - 6525999$ } 1 pt

$= 162474002$ (162,000,000)

1.c. (2pts) Also, passenger plates can not start with BS, DR, FC, FK, FU, HS, MD, PP, PU, SN, UN, VD, VP, and VT. How many are left?

ANSWER: Each two letter combination rules out a sequence of
$= 1 \times 1 \times 10 \times 26 \times 10 \times 10 \times 10$
$= 260000$ plates
There are 14 banned combinations. That removes
$= 260000 \times 14$
$= 3640000$ possible plates.

} 1 pt for work leading to

which leaves
$= 162474002 - 3640000$
$= 158834002$ (159,000,000) possible passenger plates

} 1 pt for solving

(Again, don't take off points for here for errors made in 1.a. or 1.b. provided the rest of the work is correct)

1.d. (2pts) Is this a good system or not? Why?

Use your judgement. Award 2 points if the student supports their answer with logical reasons that show the student has thought about the question. For example, the student has decided upon a reasonable criteria with which to judge the system or has identified a source of value or utility in the system.

Award 1 point for answers that are poorly supported, flippant, or non-sensical.

2.a. (3pts) How many different types of burritos can you make?

Answer: To correctly answer question 2, students have to carefully read the entire menu. The side bar establishes the following facts about burritos:

1. pinto or black bean = can't have both
2. peppers and onions instead of beans = can't have any type of bean and fajita veg
3. cheese or sour cream = only one dairy

Also from the check list

4. meat, circle one = must have exactly one meat

1 point for correctly interpreting menu

To calculate the number of possibilities, students must first calculate the number of combinations within each category. One way of doing this is with n choose m notation. This counts the number of ways we can choose m items from a group of n.

1 point for any correct method (including counting)

$$\text{Rice} = \binom{1}{0} + \binom{1}{1} = 1 + 1 = 2$$

$$\text{Beans and fajita veg} = \binom{3}{0}\binom{3}{1} = 1 + 3 = 4$$

$$\text{Meat} = \binom{5}{1} = 5$$

$$\text{Salsa} = \binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 1+4+6+4+1 = 16$$

$$\text{Dairy} = \binom{2}{0} + \binom{2}{1} = 1 + 2 = 3$$

$$\text{Guacamole} = \binom{1}{0} + \binom{1}{1} = 1 + 1 = 2$$

$$\text{Lettuce} = \binom{1}{0} + \binom{1}{1} = 1 + 1 = 2$$

Alternatively, students could look at each sub item and calculate 2 combinations for it (order or do not order) with the above restrictions.

Rice = 2

Beans and fajita veg = 4

Meat = 5

Salsa = $2 \times 2 \times 2 \times 2 = 2^4 = 16$

Dairy = 3

Guac = 2

lettuce = 2

2a (continued)

Finally, students need to take the product of these combinations to get the total number of possibilities

Total # of burritos = $2 \times 4 \times 5 \times 16 \times 3 \times 2 \times 2$ } 1 pt

$= 7680$

2.b. How many different burritos can you make if you choose no more than one option from each class?

Answer: This means that we can only choose 0 or 1 options from each class. We can adapt our results from part a accordingly. Also only the requirements to choose exactly 1 meat and not beans and fajita veg affect us. } 1 point for correctly interpreting rules in this situation

Rice = $\binom{1}{0} + \binom{1}{1} = 2$

Beans and fajita veg = $\binom{3}{0} + \binom{3}{1} = 4$

Meat = $\binom{5}{1} = 5$

Salsa = $\binom{4}{0} + \binom{4}{1} = 5$

Dairy = $\binom{2}{0} + \binom{2}{1} = 3$

Guac = $\binom{1}{0} + \binom{1}{1} = 2$

Lettuce = $\binom{1}{0} + \binom{1}{1} = 2$

} 1 point for any correct method

Again we take the product of the combinations to get total possibilities.

Total burritos = $2 \times 4 \times 5 \times 5 \times 3 \times 2 \times 2$ } 1 point

$= 2400$

2.c. (2pts) Your friend wants steak but no salsa, what is the probability that you guess everything else right?

ANSWER: We can reuse our calculations from part a (but not part b). We just have to adjust the meat and salsa categories. They each now have only 1 possibility (because we're 100% certain to pick steak and no salsa).

Step 1: Calculate the total number of burritos that fit our description.

$$\text{Burritos} = 2 \times 4 \times \underset{\text{meat}}{1} \times \underset{\text{salsa}}{1} \times 3 \times 2 \times 2 \qquad \text{(1pt)}$$

$$= 96$$

Step 2: Calculate the probability of success. Only 1 burrito will be correct. The probability that we pick it out of 96 possible burritos is:

$$P(\text{Correct burrito} \mid \text{steak + no salsa}) = \frac{1}{96} \text{ or } \approx 0.0104$$ } 1pt for dividing to get prob

2.d. How many different vegan meals can you create at Chipotle?

ANSWER: From the side bar we see that there are multiple types of meals } ½ point for noting

We can treat these separately, but the simplest way to do this problem is to group them as Chipotle does:

Burritos and Bowls
Tacos
Salads

2d (continued)

First, we calculate the total number of vegan possibilities in each group. We must pay attention to what options are available for each group.

1 pt for any correct method

In each case, to be vegan the meal must choose veg for meat, no sour cream, no dairy, and no pinto beans (but DON'T take off if the student doesn't realize about pinto beans)

Burritos & Bowls

Burrito vs. Bowl = $\binom{2}{1} = 2$ (must choose one or other)

Rice = $\binom{1}{0} + \binom{1}{1} = 2$

Beans + fajita veg = $\binom{2}{0} + \binom{2}{1} = 3$ (no pintos)

Meat = 1 (MUST be veg)

Salsa = $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 16$

Dairy = 1

Guac = $\binom{1}{0} + \binom{1}{1} = 2$

Lettuce = $\binom{1}{0} + \binom{1}{1} = 2$

Total = $2 \times 2 \times 3 \times 1 \times 16 \times 1 \times 2 \times 2$

= 768

Tacos

Crispy Corn vs. Soft Corn vs. Soft flour = $\binom{3}{0} + \binom{3}{1} = 4$

Meat = 1

Salsa = $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 16$

Dairy = 1

Lettuce = $\binom{1}{0} + \binom{1}{1} = 2$

Total = $4 \times 1 \times 16 \times 1 \times 2$

= 128

2d (continued 2)

Salads

Only the vegetarian salad applies, so

Beans = 1 (no pinto)

Meat = 1 (Veg)

Salsa = $\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4} = 16$

Cheese = 1 (no cheese)

Total = $1 \times 1 \times 16 \times 1$

$= 16$

note* accept 32 and 64 if the students assumed that you could hold the vinagarette and lettuce

Now we add up all the types of meals

Total vegan meals = $768 + 128 + 16$ } $\frac{1}{2}$ point

$= 912$

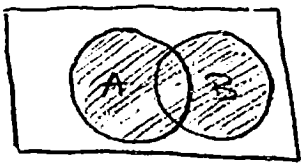
Note: If a student decided to use the same methods to factor in the different extras, drinks, or drink sizes that can be included in a meal assign them 1 bonus point ☺

Note Note: the above does not apply to 2a-c because a drink/extra is not part of a burrito

Note Note: No meal (i.e, nothing) does not count as a burrito nor a vegan meal by definition.

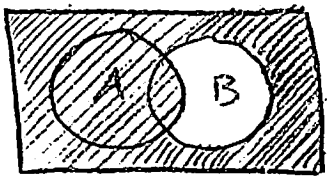
3. Draw a venn diagram of each set, then simplify where possible
(½ point each: ¼ for diagram, ¼ for simplification)

a. $A \cup B$



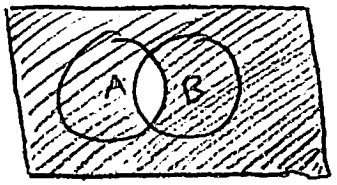
no simplification

b. $A \cup B^c$



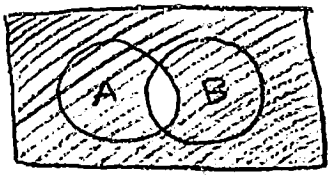
no simplification
but accept $(A^c \cap B)^c$

c. $A^c \cup B^c$



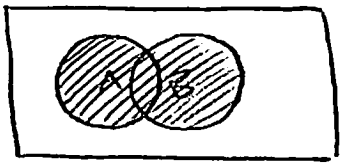
$(A \cap B)^c$

d. $(A \cap B)^c$



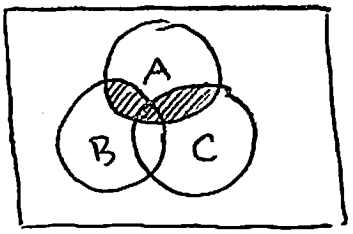
$A^c \cup B^c$

e. $(A^c \cap B^c)^c$



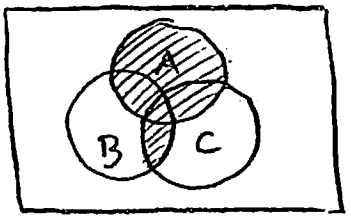
$A \cup B$

f. $(A \cap B) \cup (A \cap C)$



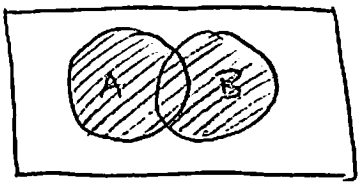
$A \cap (B \cup C)$

g. $(A \cup B) \cap (A \cup C)$



$A \cup (B \cap C)$

h. $(A \cap A) \cup (B \cup B)$



$A \cup B$