

If you choose an answer to this question at random, what is the chance you will be correct?

A) 25%

B) 50%

C) 60%

D) 25%

Stat310

Equally likely outcomes

Hadley Wickham

1. Why bother?
2. Assumptions and formal definition
3. Back to the governorator
4. Counting revision
5. Worked examples

Motivation

- Start small!
- Simplest useful case is where all outcomes are equally likely
- (Not that all events are equally likely!)

Assumption:
all outcomes are
equally likely

Implication: our
sample space is finite

Equally likely outcomes

$$S = \{o_1, o_2, o_3, \dots, o_m\}$$

$$P(\{o_i\}) = p = 1 / m$$

$$\text{If } A \subset S, P(A) = |A| / |S| = |A| / m$$

So all we have to do is count!
(usually both the sample space and the event)

Let's try and use this to solve a real problem

To the Members of the Assembly

Did this message arise from chance?

For some time now I have lamented the fact that major issues are overlooked while many unnecessary bills come to me for consideration. Water reform, prison reform, and health care are major issues my Administration has brought to the table, but the Legislature just kicks the can down the alley.

Yet another legislative year has come and gone without the major reforms Californians overwhelmingly deserve. In light of this, and after careful consideration, I believe it is unnecessary to sign this measure at this time.

Sincerely,

Arnold Schwarzenegger

Big question

How do we make this a random experiment? i.e. how do we make it repeatable?

We can't lock Arnie in a room and force him to write multiple vetoes, so we're going to have to cheat and come up with some way of reproducing vetos - this is a statistical model.

Your turn

How could we create a veto randomly?
What are the component pieces? How
could we replace or rearrange them?

Take two minutes and brainstorm as
many ways as you can think of.

Pieces

Paragraphs

Lines

Words

Letters

Probability models

- Chose letters at random
- Chose words at random
- Randomly reorder lines
- Randomly reorder words
- Randomly reorder words within lines
- Randomly place line breaks

Counting revision



2

3

Principles

- 0th law: Add exclusive options
- 1st law: Multiply successive choices
- 2nd law: Overcounting by a uniform multiplicative factor can be corrected by dividing by this factor

<http://bit.ly/zfgTkx>

Your turn

How many distinct permutations of mississippi are there?

How many ways can you arrange 12 people into 3 equally sized groups?

Tools

- Multiplication principle
- Sampling with replacement (n^r)
- Sampling without replacement
 - Permutation: $n(n - 1)\dots(n - r + 1) = n! / (n - r)!$
 - Combination: permutation / $r!$

Tips

- You should be able to derive any of these results!
- Cancel factorials if big.
- Some times simpler to think about the ones left out.

Your turn

Go back to the 6 ways of generating a new veto. Which counting method will you use for each?

(Chose letters at random, chose words at random, randomly reorder lines, randomly reorder words, randomly reorder words within lines, randomly place line breaks)

My ideas

Sampling with replacement: letters and words

Permutations: lines, words, words within lines

Combinations: random line breaks

Worked examples

To get probability

Count number of outcomes in sample space.

Count number of outcomes in event of interest.

Divide.

With replacement

If the first letter of each line was sampled at random from the alphabet, what's the probability that the letter spell "I fuck you"?

(Challenge: what's the probability they spell any offensive message)

Your turn

If we wanted to select words at random, where could we get them from?

You have 30 seconds to brainstorm 5 places. Work with your neighbours.

Letter	Dictionary	Gutenberg
c	85	35
f	29	38
i	37	63
k	9	7
o	33	63
u	70	14
y	3	16

Pretending there are
1,000 words in each

Permutations

Lines

Words

Words within lines

Permuting lines

i, c, f, k, o, u₁, u₂, y

Permuting words

What information do we need?

First letter	Frequency
i	6
c	8
f	3
k	1
o	3
u	2
y	2
Total	87

Line	Words	Starting with letter
i	9	1
f	16	2
u	13	1
c	15	1
k	6	1
y	13	2
o	14	1
u	8	1

Combinations

Placement of line breaks

I am returning Assembly Bill 1176 without my signature. For some time now I have lamented the fact that major issues are overlooked while many unnecessary bills come to me for consideration. Water reform, prison reform, and health care are major issues my Administration has brought to the table, but the Legislature just kicks the can down the alley. Yet another legislative year has come and gone without the major reforms Californians overwhelmingly deserve. In light of this, and after careful consideration, I believe it is unnecessary to sign this measure at this time.

Homework

Up on website. Due Thursday 5pm, in DH mail room. 10% penalty if one day late. Not accepted after that.

Should be stapled, and your handwriting needs to be legible. Graders will penalise if not.

No more than 3 significant figures.

Practice problems will have solutions once I get a TA.

Next time

Working with conditional probability.

A family has two children. One of them is a boy. What's the probability that the other is a girl?