

Stat310

Conditional probability

Hadley Wickham

1. Amazon and homework updates
2. Basic properties of probability
3. Conditional probability
4. Law of total probability
5. Independence

Month to Date

Last updated: January 16, 2012

You are currently enrolled in our **Performance** Fee Structure. [Learn more](#)

[View your payment history](#)

Earnings Summary		Full report
Total items shipped		27
Referral Rate		N/A
Advertising Fees		\$91.08
TOTAL EARNINGS THIS MONTH		\$91.08
Glossary		Click here to view the referral rate tier chart.

Orders Summary		Full report
Ordered items		37
Clicks		64
Conversion		57.81%
Glossary		

Books					
Mathematical Statistics with Applications	10.94%	64	7	0	7
Mathematical Statistics with Applications	9.38%	64	6	0	6
An Introduction to the Solar System	N/A	0	0	1	1
C Programming Language (2nd Edition)	N/A	0	0	1	1
C: A Reference Manual (5th Edition)	N/A	0	0	1	1
Case Histories in International Politics (6th Edition)	N/A	0	0	1	1
Communication Systems Engineering (2nd Edition)	N/A	0	0	1	1
Computer Systems: A Programmer's Perspective (2nd Edition)	N/A	0	0	1	1
Creating Capabilities: The Human Development Approach	N/A	0	0	1	1
Discrete-Time Signal Processing (3rd Edition) (Prentice Hall Signal Processing)	N/A	0	0	1	1
Engineering Mechanics, Statics	N/A	0	0	1	1
Intermediate Microeconomics: A Modern Approach (Eighth Edition)	N/A	0	0	1	1
Minds, Brains, and Computers: An Historical Introduction to the Foundations of Cognitive Science (Blackwell Philosophy Anthologies)	N/A	0	0	1	1
Pattern Recognition and Machine Learning (Information Science and Statistics)	N/A	0	0	1	1
The Art of Multiprocessor Programming	N/A	0	0	1	1

Homework 0

Check your grades on owlspace.

3 = read future instructions more carefully

Obviously you don't get them back.

Facts
about
your
five
facts

Facts
about
your
five
facts

15 CS majors

Facts
about
your
five
facts

15 CS majors
11 from China

Facts
about
your
five
facts

15 CS majors
11 from China
6 acrostics

Facts	15 CS majors
about	11 from China
your	6 acrostics
five	5 speak 3+ languages
facts	

Facts	15 CS majors
about	11 from China
your	6 acrostics
five	5 speak 3+ languages
facts	4 play rugby

Facts	15 CS majors
about	11 from China
your	6 acrostics
five	5 speak 3+ languages
facts	4 play rugby
	3 are twins

Facts
about
your
five
facts

15 CS majors

11 from China

6 acrostics

5 speak 3+ languages

4 play rugby

3 are twins

2 have black belts in
taekwondo

Facts
about
your
five
facts

15 CS majors

11 from China

6 acrostics

5 speak 3+ languages

4 play rugby

3 are twins

2 have black belts in
taekwondo

1 limerick

Homework help

Tuesday and Wednesday, 5-6pm

Mech Lab 251 (may change next week)

Remember: it's informal!

Probability

Three axioms:

Probability is a function, P , that satisfies these conditions:

1. $P(A) \geq 0$, for all $A \subset S$
2. $P(S) = 1$
3. $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \emptyset$

Any function that satisfies these axioms is called a probability function.

Basics

Recall, or
prove if asked

$$P(A') = 1 - P(A)$$

If $A \subset B$, then $P(A) \leq P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

You should be able to see these from a venn diagram, and prove them using set theory and the axioms of probability.

Performing a proof

What am I trying to prove? (**NTS**)

What can I use to prove it? (**GIVEN**)

Sequence of logical steps from assumptions to hypothesis. Each step justified with a tool.

Am I finished? (\square)

Prove that

$$P(A') = 1 - P(A)$$

Intuition?

Tools?

Your turn

Prove that if $A \subset B$, then $P(A) \leq P(B)$

Work with your neighbour.

Conditional probability

Your turn

A family has two children. They do not have two girls. What is the probability they have a boy and a girl?

What is a paradox?

A paradox is a statement or group of statements that leads to a contradiction or a situation which **defies intuition**.

(<http://en.wikipedia.org/wiki/Paradox>)

Our statistical intuition is often wrong, so we need a careful mathematical process to check

Process

Convert word problem to mathematical problem (Write down what is known)

Define sample space and events of interest. (What do we need to figure out)

Use tools of probability to go from what is given to what is needed.

We need tools for
dealing with conditional
probabilities

Conditional probability

Definition. Sample space interpretation.

Is the conditional probability a probability function?

Multiplication “rule”

Example

What is the probability that a randomly picked person out of this class is pregnant in a years time?

(~1.7 million pregnancies to girls aged 20-24, ~30 million girls)

Process

Convert word problem to mathematical problem (Write down what is known)

Define sample space and events of interest. (What do we need to figure out)

Use tools of probability to go from what is given to what is needed (complements? intersection? conditioning?)

Law of total probability

Example

Randomly pick a guy from this class.
What is the probability he gets a girl
pregnant in the next year?

How might we want to partition the
sample space?

Data

$$P(\text{no sex}) = 0.2$$

$$P(\text{gay sex}) = 0.05$$

$$P(\text{no contraception}) = 0.18 * 0.75$$

$$P(\text{uses condom}) = 0.71 * 0.75$$

$$P(\text{withdrawal}) = 0.11 * 0.75$$

$$P(\text{pregnant} \mid \text{no contraception}) = 0.85$$

$$P(\text{pregnant} \mid \text{use condom}) = 0.15$$

$$P(\text{pregnant} \mid \text{withdrawal}) = 0.27$$

Independence

If A and B are **independent**, then
 $P(A \cap B) = P(A) P(B)$

This implies $P(A | B) = P(A)$ - the fact that B occurred does not give us any information about A (and vice versa)

Events are **mutually independent** if ...

Your turn

Are **mutually exclusive** events **mutually independent**? Why/why not?

When is an event independent of itself?

When is A independent of A ?

Another proof

Assume that A and B are independent.
Show that A and B' are independent.

In homeworks and exams, correctly turning a word problem into a math problem will get you a few points. Writing down possible strategies will get you a few more.