STAT310 01/17

①

A Ā

NTS $P(\bar{A}) = 1 - P(A)$

i.e. $P(\bar{A}) + P(A) = 1$

GIVEN P is a probability function
A is a set

result. reason

$P(\bar{A}) + P(A) = P(A \cup \bar{A})$ axiom 3 $A \cap \bar{A} = \phi$

$= P(S)$ set theory

$= 1 \ \square$ axiom 2

we're done.

CONDITIONAL PROBABILITY.

Motivation: 4 equally likely ~~events~~ outcomes: BB, BG, GB, GG.

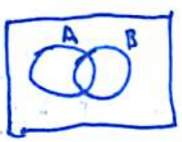
$S = \{BB, BG, GB\}$

$B = \{BG, GB\}$

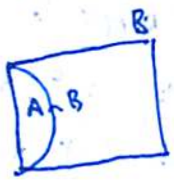
$P(B) = 2/3$.

DEFINITION: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ $P(B) > 0$

A given B



$\Rightarrow$



②

Let ~~f(A)~~ $f(A) = P(A \mid B)$ $P(B) > 0$

is f a probability function?

(1) $f(A) = P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} \begin{matrix} \geq 0 \\ > 0 \end{matrix}$

$\Rightarrow f(A) \geq 0$ ✓

(2) $f(S) = P(S \mid B) = \dfrac{P(S \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$ $(P(B) > 0)$ ✓

(3) ~~f(A∪B) = P(A∪B)~~

$f(X \cup Y) = P((X \cup Y) \mid B) = \dfrac{P((X \cup Y) \cap B)}{P(B)}$ def.

$= \dfrac{P((X \cap B) \cup (Y \cap B))}{P(B)}$ set theory

$= \dfrac{P(X \cap B)}{P(B)} + \dfrac{P(Y \cap B)}{P(B)}$ ✓

$= P(X \mid B) + P(Y \mid B) = f(X) + f(Y) \ \square$

So conditional prob is a probability $\Rightarrow$

$P(\bar{A} \mid B) = 1 - P(A \mid B)$

$P(A \mid B) \leq P(X \mid B)$ if $A \subset X$

~~P(X∪Y|B)~~

$P((X \cup Y) \mid B) = P(X \mid B) + P(Y \mid B) - P(X \cap Y \mid B)$

MULTIPLICATION "RULE"

$P(A \cap B) = P(A \mid B)\, P(B)$

showed math.

EXAMPLE

$F = \{\text{female}\}$ $\quad P(F) = 0.5$

$R = \{\text{pregnant}\}$ $\quad P(R) = ?$

$$P(R) = P(R|F) + \underbrace{P(R|\bar{F})}_{=0} = P(R|F)\,P(F)$$

$$= \frac{1.7}{36} \cdot 0.5 = 0.0024.$$

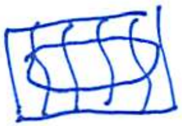
LAW OF TOTAL PROBABILITY

$\{P_i : i = 1, \dots, n\}$ be a partition of S

Then $P(A) = \sum_{i=1}^{n} P(A|P_i)\,P(P_i)$

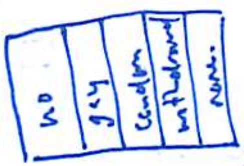
$$= \sum_{i=1}^{n} P(A \cap P_i)$$

Intuition:



add up little pieces to get big piece.

EXAMPLE



$GP = \{\text{gets girl pregnant}\}$

$P(GP) = P(GP \mid \text{no sex})\,P(\text{no sex}) +$
$P(GP \mid \text{gay sex})\,P(\text{gay sex}) + \dots$
$\dots +$
$P(GP \mid \text{none})\,P(\text{none})$.

$= 0 + 0 + 0.85 \cdot 0.15 + 0.15 \cdot 0.71 + 0.27 \cdot 0.11$

$= 0.264$.

INDEPENDENCE

A, B, C are mutually indep if:

$P(A \cap B) = P(A)P(B)$ $\quad P(A \cap C) = P(A)P(C)$

$P(B \cap C) = P(B)P(C)$

AND

$P(A \cap B \cap C) = P(A)P(B)P(C)$

and so on if more than 3 sets.

Note: why do we define w/ intersection and not condition?

NTS $P(A \cap \bar{B}) = P(A)P(\bar{B})$

GIVEN $P(A \cap B) = P(A)P(B)$
(and P is a prob. function, A & B are sets)

Two approaches (50% inspiration, 50% hard work)

$$P(A \cap \bar{B}) = P(\bar{B}|A)\,P(A)$$
$$= (1 - P(B|A))\,P(A)$$
$$= (1 - P(B))\,P(A)$$
$$= P(\bar{B})\,P(A) \quad \square$$

$$P(A \cap \bar{B}) = 1 - P(\bar{A} \cup B)$$
$$= 1 - [P(\bar{A}) + P(B) - P(\bar{A} \cap B)]$$
$$= P(A) - P(B) + P(\bar{A} \cap B)$$
$$= P(A) - P(A \cap B) + \underbrace{[P(\bar{A} \cap B) + P(A \cap B) - P(B)]}_{=0}$$
$$= P(A) - P(A)P(B) = P(A)(1 - P(B)) = P(A)P(\bar{B}) \quad \square$$