# Stat310

## Discrete random variables

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1. Transformation and expectation
2. Binomial distribution
3. Discrete uniform distribution
4. Poisson distribution
5. Negative binomial distribution
6. Feedback

# Homework help sessions

From now on in Duncan 113.

Tuesday & Wednesday 5-6pm.

Tuesday: Garrett
Wednesday: Me

# Expectation

# Your turn

| x | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| f(x) | 0.2 | 0.1 | 0.3 | 0.1 | 0.3 |

Let X be a discrete random variable with pmf f as defined above.

Write out the pmfs for:

$A = X + 2$ $B = 3X$ $C = X^2$ $D = 0 * X$

# Expectation of a function

$$E(g(X)) = \sum_{x \in S} f(x) g(x)$$

expectation as a function
little x vs. big x

# Variance

$Var(X) = E[ (X - E[X])^2) ] = ?$

Measures the spread. More intuition next week.

# Binomial

http://demonstrations.wolfram.com/BinomialProbabilityDistribution/

# Discrete uniform

# Discrete uniform

Equally likely events labelled with integers from a to b.  $n = (b - a + 1)$

$f(x) = 1/n \quad x = a, ..., b$

$X \sim \text{DiscreteUniform}(a, b)$

# Examples

Rolling a dice

Spinning a wheel of fortune

Anything with equally likely outcomes...

# Your turn

Intuitively, what do you expect the mean to be?

How would you work it out? What sum do you need to evaluate?

Intuitively how do you expect the variance (spread) to change as the parameters change?

# Wolfram alpha

```
sum_(x = a)^b x / (b - a + 1)
```

```
sum_(x = a)^b x^2 / (b - a + 1)
simplify (-a + 2 a^2 + b + 2 a b + 2 b^2)/6 - ((a
+ b)/2)^2
```

```
sum_(x = a)^b (x - (a + b)/2)^2 / (b - a + 1)
```

```
_ = underneath
^ = above
```

# Poisson distribution

3.2.2 p. 119

# Conditions

X = Number of times some event happens

(1) If number of events occurring in non–overlapping times is **independent**, and

(2) probability of exactly one event occurring in short interval of length h is $\propto$ λh, and

(3) probability of two or more events in a sufficiently short internal is basically 0

# Poisson

X ~ Poisson(λ)

Sample space:
positive integers

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

λ ∈ (0, ∞)

λ is a rate - the expected number of events per unit time

Arises as limit of binomial distribution when np is fixed and n → ∞ = law of rare events. (See wikipedia for proof if interested) http://demonstrations.wolfram.com/IndividualAndCumulativePoissonProbabilities/

http://www.wolframalpha.com/input/?i=poisson+distribution%2C+mu+%3D+1

# Examples

Number of alpha particles emitted from a radioactive source

Number of calls to a switchboard

Number of eruptions of a volcano

Number of points scored in a game

# Examples

On average there are a million earthquakes per year. What is the distribution of earthquakes in the next year? What is the distribution in the next ten years? In the next minute?

A switchboard an average of 15 calls per hour. What's the distribution of the number of calls? How many operators do they need?

# Your turn

How do we check that this is a valid pmf?

If $X \sim \text{Poisson}(\lambda)$, what is $E(X)$ ?

# Wolfram alpha

```
sum_(x = 0)^oo exp(-lambda) lambda^x / x!
sum_(x = 0)^oo x exp(-lambda) lambda^x / x!

oo = infinity
lambda = λ
exp(x) = e^x
```

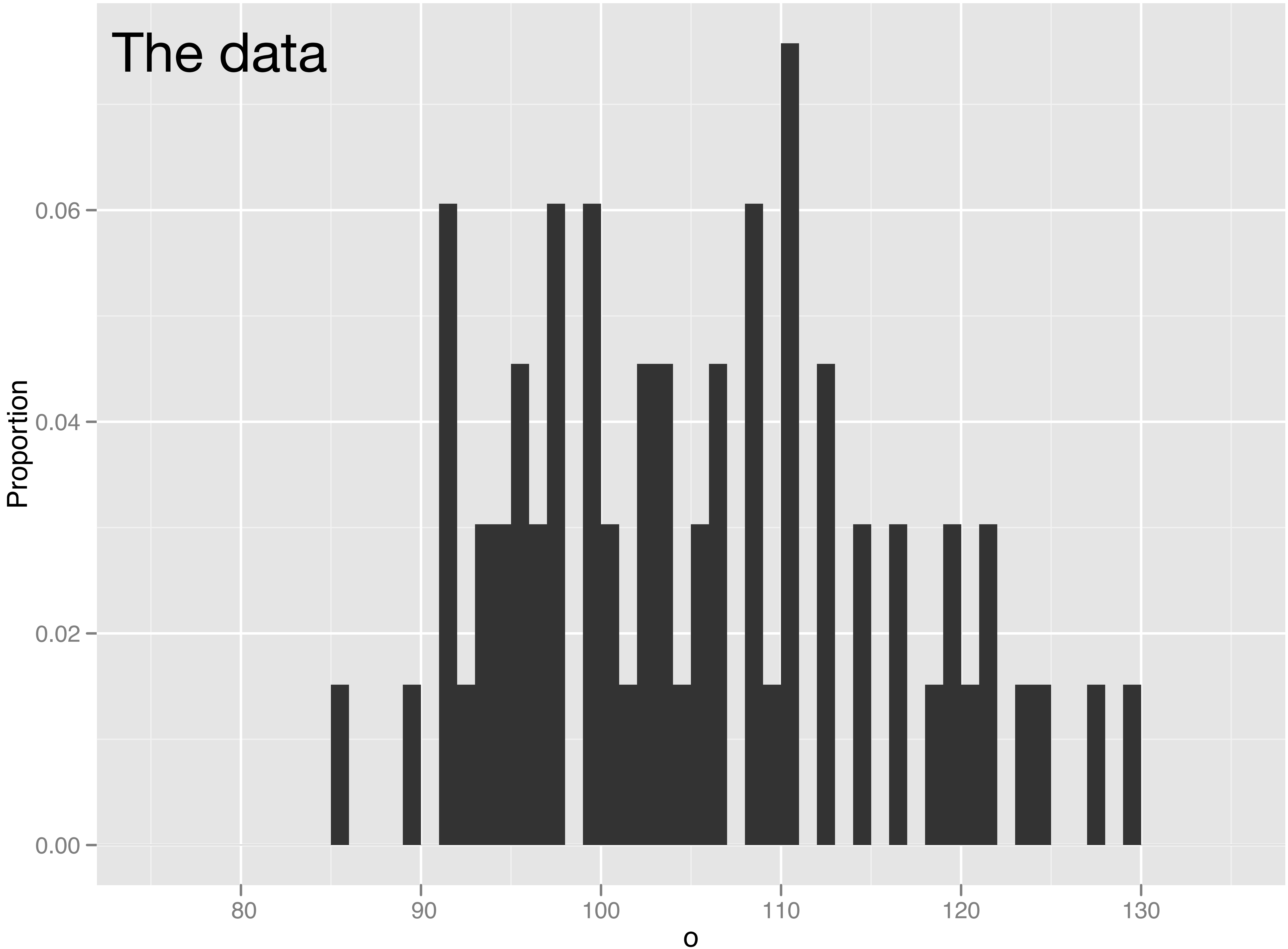
# Basketball

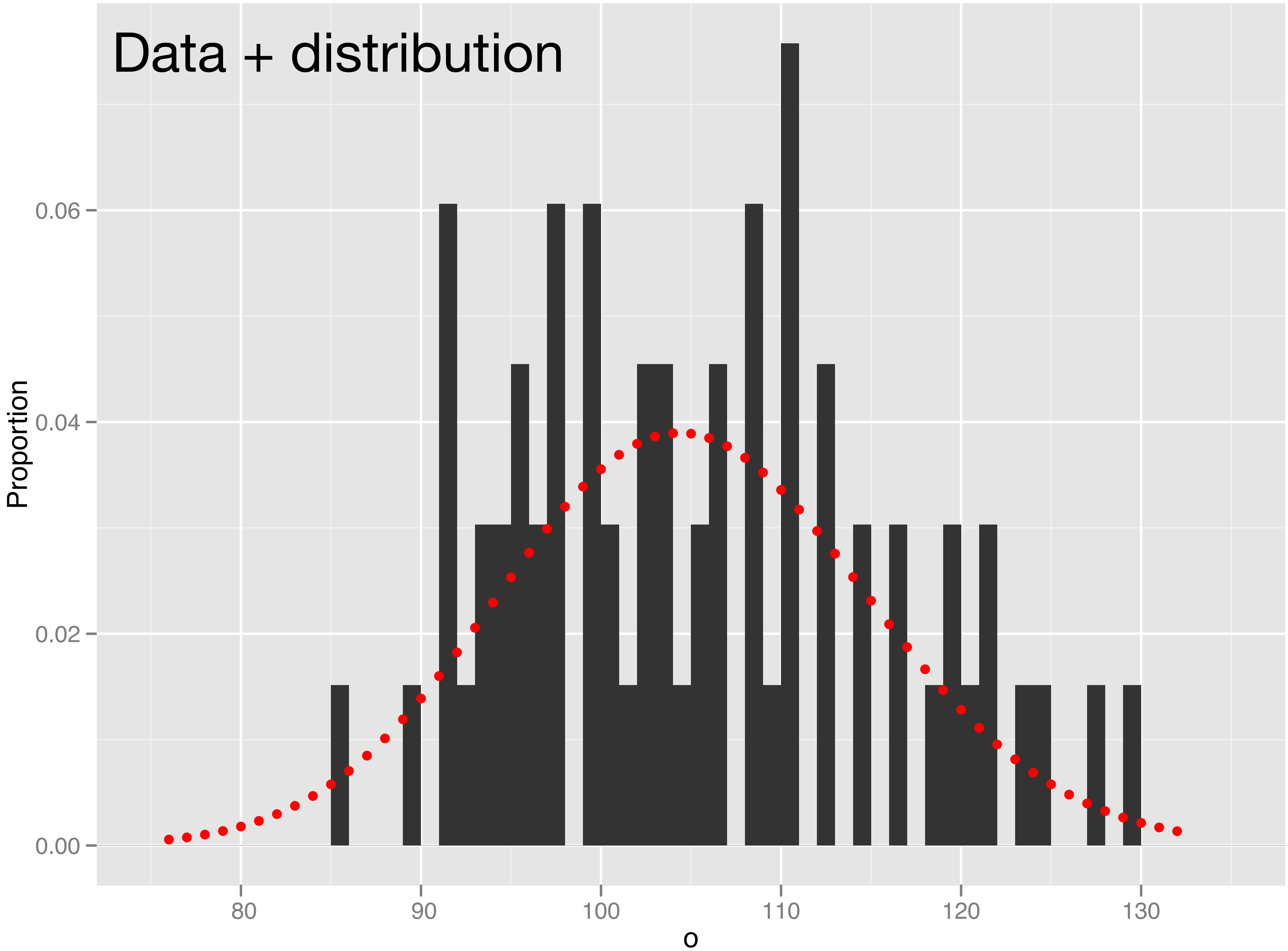
Many people use the poisson distribution to model scores in sports games. For example, last season, on average the Houston Rockets scored 109.5 points per game, and had 106.2 points scored against them. So:

$O \sim \text{Poisson}(109.5)$
$D \sim \text{Poisson}(106.2)$

How can we check if this is reasonable?

http://www.databasebasketball.com/teams/teamyear.htm?tm=LAL&lg=n&yr=2010

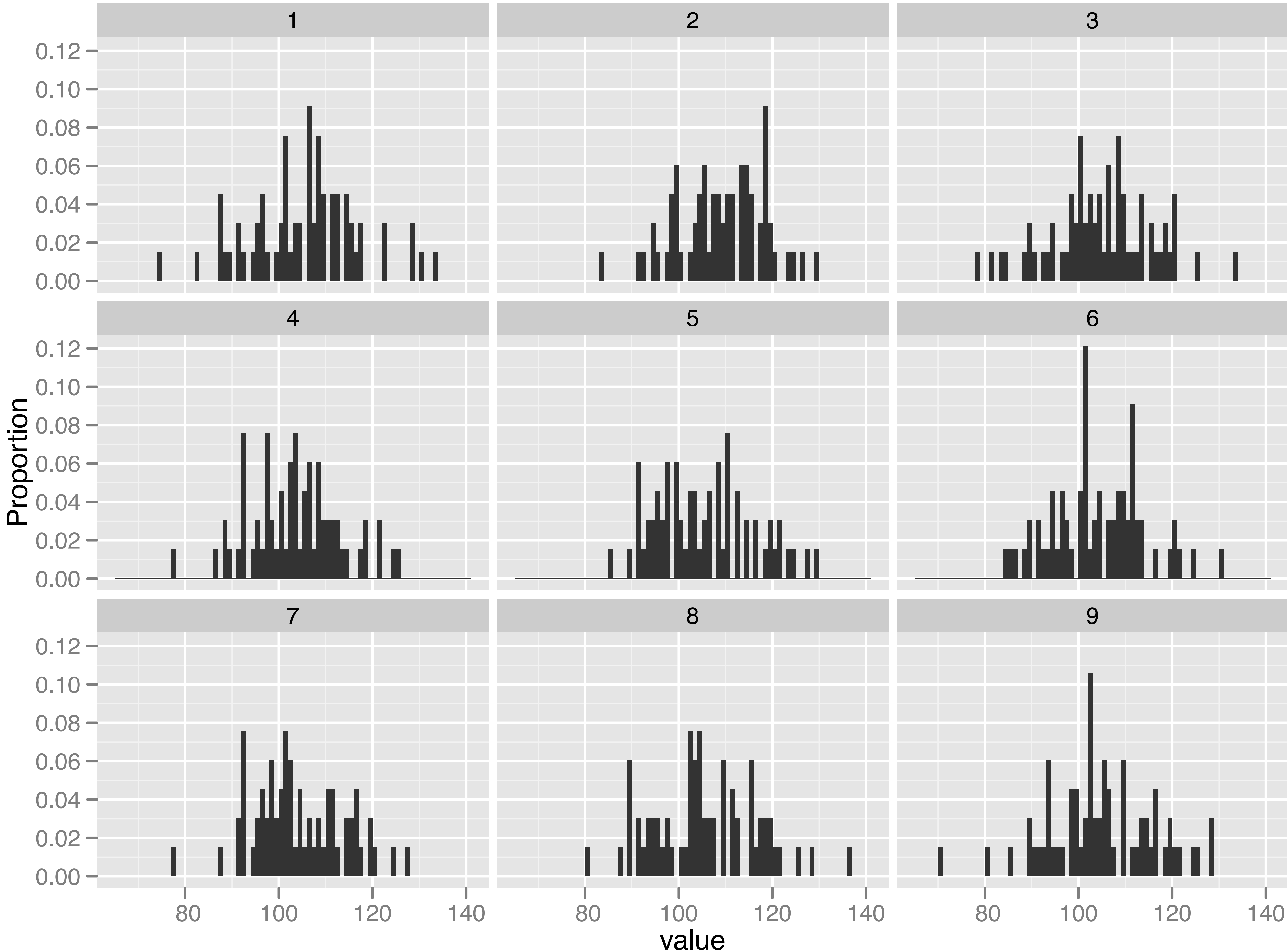

The data
Proportion
0.06
0.04
0.02
0.00
80
90
100
110
120
130
o


Data + distribution
Proportion
0.06
0.04
0.02
0.00
80
90
100
110
120
130
o

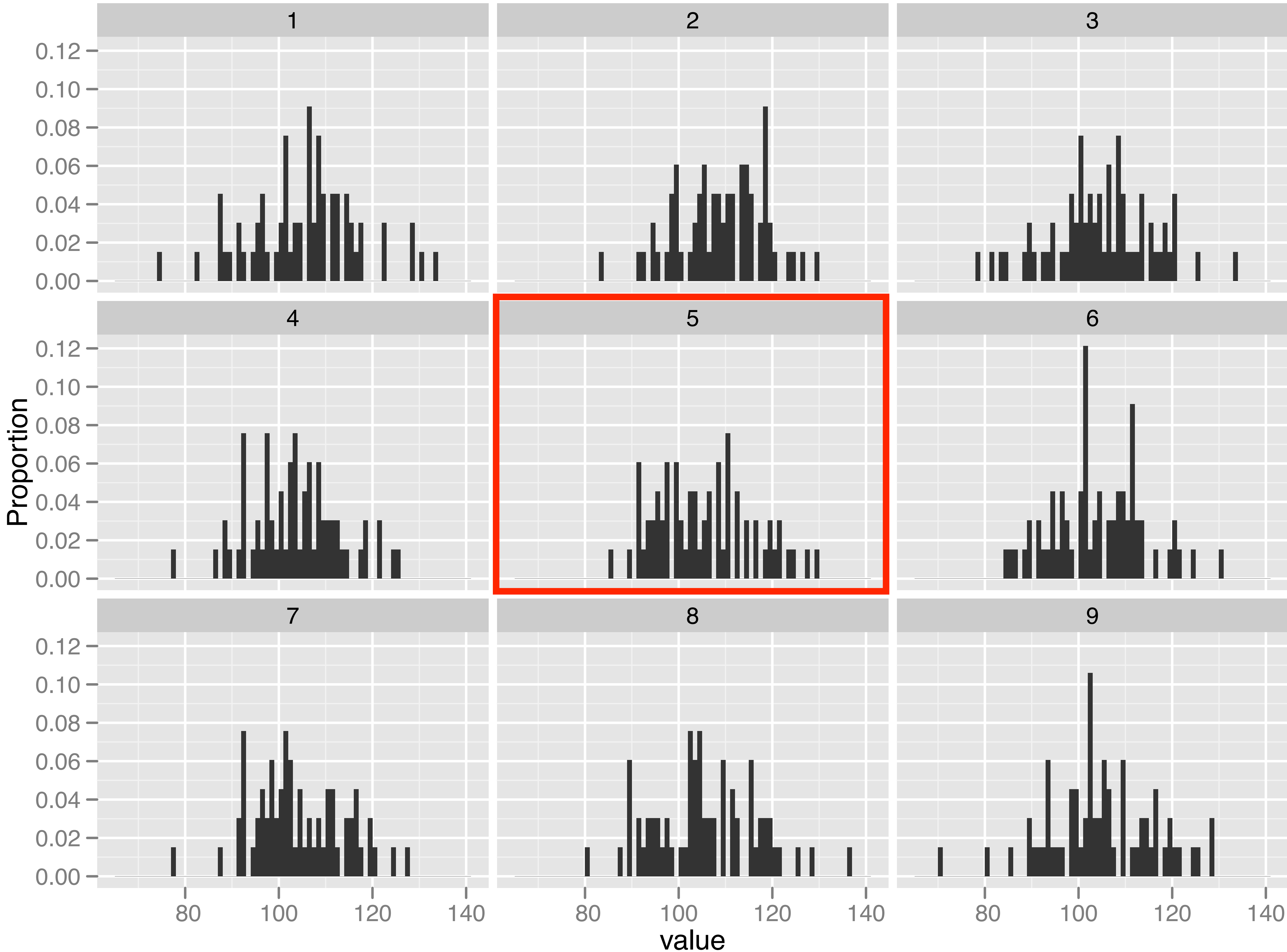
# Simulation

Comparing to underlying distribution works well if we have a very large number of trials. But only have 66 here.

Instead we can randomly draw 66 numbers from the specified distribution and see if they look like the real data.



1
2
3
4
5
6
7
8
9
Proportion
0.12
0.10
0.08
0.06
0.04
0.02
0.00
80
100
120
140
value




Thursday, January 26, 12

# But

The distribution can't be poisson.  Why?

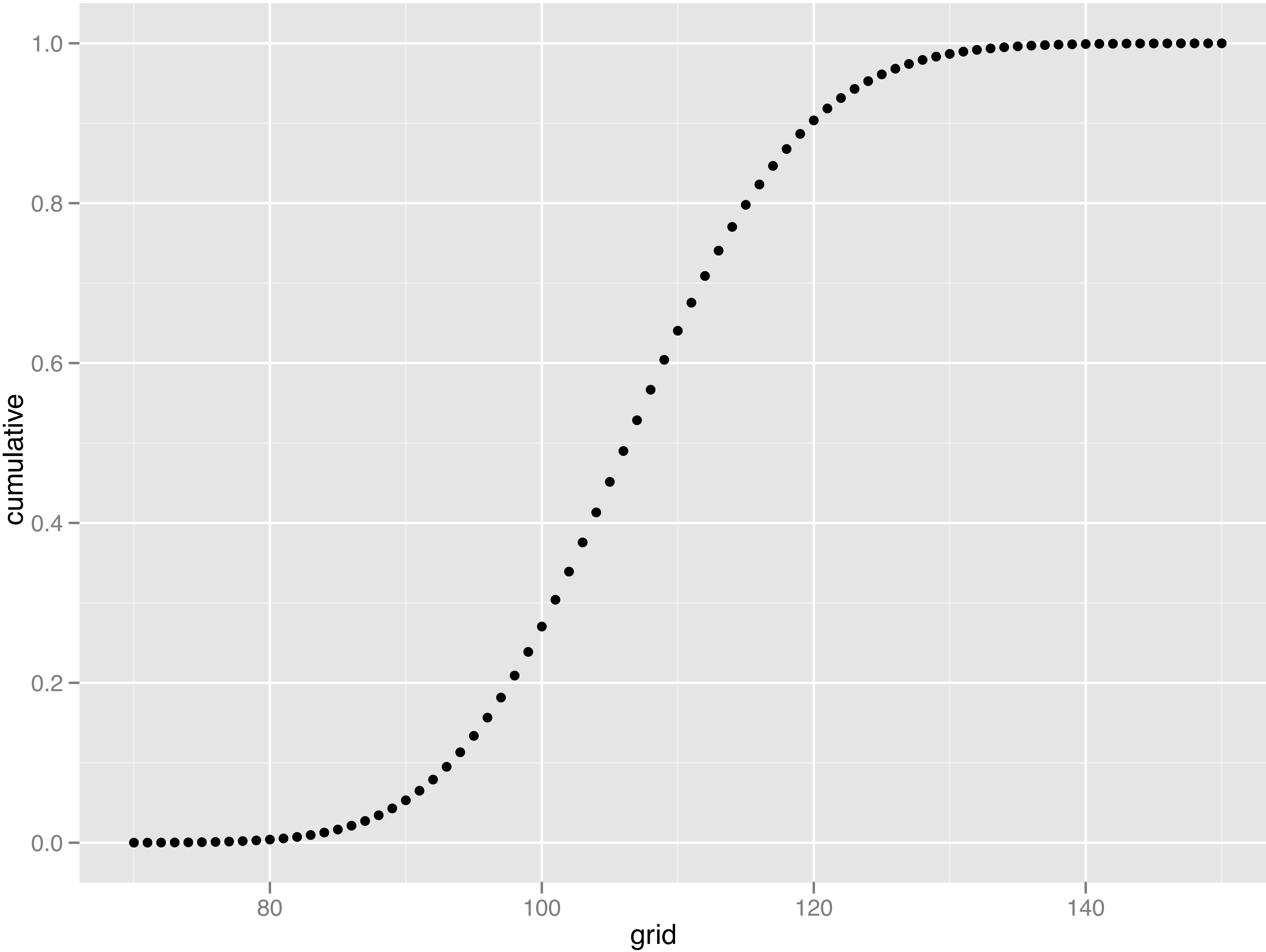
# Example

O ~ Poisson(109.6). D ~ Poisson(101.8)

On average, do you expect the Lakers to win or lose? By how much?

What's the probability that they score exactly 100 points?

What's the probability they score over 100 points? (How could you work this out?)



1.0
0.8
0.6
0.4
0.2
0.0
cumulative
80
100
120
140
grid

# Negative binomial

# Negative binomial

X ~ How many successes from independent Bernoulli(*p*) trials, until *r* failures. ($0 < p < 1$, $r = 1, 2, \ldots$).

Support = 0, 1, 2, ...

$$f(x) = \binom{x + r - 1}{x} (1 - p)^r p^x$$

# Examples

How many tails until I get 5 heads?

Door-to-door salesman: how many houses does he have to visit until he sells 10 vacuum cleaners?

How many successful shots before Kobe Bryant misses?

Mean

$$r\frac{p}{1-p}$$

Variance

$$r\frac{p}{(1-p)^2}$$

# Your turn

In the 2008–09 season, Kobe Bryant had a 0.86 success rate at field goals.

On average, how many attempts would he need to make until he made 10 shots?

What's the probability it takes more than 12 attempts?

# Feedback