X	A	B	C	D	Probability
−1	1	−3	1	0	0.2
0	2	0	0	0	0.1
1	3	2	1	0	0.3
2	4	6	2	0	0.1
3	5	9	3	0	0.3

(1)

C

c	$f_C(c)$
0	0.1
1	0.5
2	0.1
3	0.3

D

d	$f_D(d)$
0	1

$E : \text{pmf} \to \mathbb{R}$ $\quad \left(\frac{d}{dx} \ (\mathbb{R} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{R}) \right)$ (2)

if you don't get a number then something has gone wrong.

$$E\left((X - E(X))^2\right) = E\left(X^2 - 2X E(X) + E(X)^2 \right)$$

$$= E(X^2) + E(-2XE(X)) + E(E(X)^2)$$

$$= E(X^2) - 2E(X)E(X) + E(X)^2$$

$$= E(X^2) - E(X)^2 \quad \square$$

$$\boxed{\mathrm{Var}(X) = \frac{n^2 - 1}{12} \qquad n = b - a + 1}$$

DISCRETE RANDOM VARIABLES

DISCRETE UNIFORM

$E(X) = \frac{a+b}{2}$ $\quad Var(X) = \frac{(a-b-1)^2 - 1}{12}$

$= \frac{n^2-1}{12}$

POISSON

$E(X) = \lambda$ $\quad Var(X) = \lambda^2$

why can't points in basketball be Poisson?

* free throws - game clock isn't advanced
* can score more than 1 point at a time

EXAMPLE

$E(O-D) = E(O) - E(D)$

$= 109.6 - 101.8$

$= 7.8$

$P(O=100) = \frac{e^{-109.6} (109.6)^{100}}{100!}$

$= 0.0258$

$P(O > 100) = 1 - \sum_{i=1}^{100} P(X = x)$

$=$ sum dpois (1:99, lambda = 109.6))

$= 0.167$

NEGATIVE BINOMIAL

here making goal = failure

$p = 0.14 \quad r = 10$

$E(F) = 10 \times \frac{0.14}{0.86} = 1.62$

$\approx 10 + 1.62 = 11.62$ attempts for 10 successes

≈ 12

$P(F > 12-10) = P(F > 2)$

$= 1 - P(F \leq 2)$

$= 1 - [P(F=2) + P(F=1) + P(F=0)]$

$= 1 - \underbrace{\binom{10+2-1}{2}}_{55} 0.86^{10}\, 0.14^{2}$

$- \underbrace{\binom{10+1-1}{1}}_{10} 0.86^{10}\, 0.14^{1}$

$- \binom{10+0-1}{0} 0.86^{10}$

$= 1 - (0.548) = 0.452$ □