Stat310

Moments

Hadley Wickham

Dog sitting

- Opportunity for Mina and Lola to visit class
- But I need two dog “sitters” - email me if you are interested.

1. Feedback
2. Negative binomial
3. Moments
4. The moment generating function
5. Using the mgf
6. Summary of discrete random variables

Feedback

-- examples

-- examples

------------------------------------ engaging/energetic

-- examples

------------------------------------ engaging/energetic

-------------------- your turns

-- examples

------------------------------------ engaging/energetic

-------------------- your turns

--------------- slides

-- examples

------------------------------------ engaging/energetic

-------------------- your turns

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--- examples

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------------ homeworks

-- examples

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----------- fun/funny

-- examples

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-------------- explanations

------------ homeworks

----------- fun/funny

---------- website

-- examples

------------------------------------ engaging/energetic

-------------------- your turns

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------------ homeworks

----------- fun/funny

---------- website

------- loud

-- examples

------------------------------------ engaging/energetic

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----------- fun/funny

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------- loud

----- punctual/prompt

-- examples
------------------------------------ engaging/energetic
-------------------- your turns
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------------ homeworks
----------- fun/funny
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------- loud
----- punctual/prompt
----- accent

-- examples

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-------------------- your turns

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------------ homeworks

----------- fun/funny

---------- website

------- loud

----- punctual/prompt

----- accent

--- wolfram alpha

--- examples

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----------- fun/funny

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------- loud

----- punctual/prompt

----- accent

--- wolfram alpha

- best intro to proofs

-- examples
------------------------------------ engaging/energetic
-------------------- your turns
--------------- slides
-------------- explanations
------------ homeworks
----------- fun/funny
---------- website
------- loud
----- punctual/prompt
----- accent
--- wolfram alpha
- best intro to proofs

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------------ help sessions not helpful

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------- easier hw

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------- solutions online

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------- easier hw

------- solutions online

--- more practice problems

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--- more practice problems

--- more time in your turn

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------- solutions online

--- more practice problems

--- more time in your turn

-- your turn: sample without replacement

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-------------- ambiguous homeworks

------------ help sessions not helpful

------- easier hw

------- solutions online

--- more practice problems

--- more time in your turn

-- your turn: sample without replacement

-- your turn: stressful

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-------------- ambiguous homeworks

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-- your turn: sample without replacement

-- your turn: stressful

-- alternate b/w left & right boards

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-- your turn: sample without replacement

-- your turn: stressful

-- alternate b/w left & right boards

- poor board numbering

--------------------------- too fast

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-- your turn: sample without replacement

-- your turn: stressful

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- more scooby snacks

--------------------------- too fast

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Negative binomial

Negative binomial

X ~ How many successes from independent Bernoulli(*p*) trials, until *r* failures. (0 < p < 1, r = 1, 2, ...).

Support = 0, 1, 2, ...

$$f(x) = \binom{x+r-1}{x}(1-p)^r p^x$$

Examples

How many tails until I get 5 heads?

Door-to-door salesman: how many houses does he have to visit until he sells 10 vacuum cleaners?

How many successful shots before Kobe Bryant misses?

Mean

$$r\frac{p}{1-p}$$

Variance

$$r\frac{p}{(1-p)^2}$$

Your turn

In the 2008–09 season, Kobe Bryant had a 0.86 success rate at field goals.

On average, how many attempts would he need to make until he made 10 shots?

What's the probability it takes more than 12 attempts?

Process

Convert word problem to mathematical problem (Write down what is known)

Define sample space and events of interest. (What do we need to figure out)

Identify random variable that is closest match.

Use tools of probability to go from what is given to what is needed.

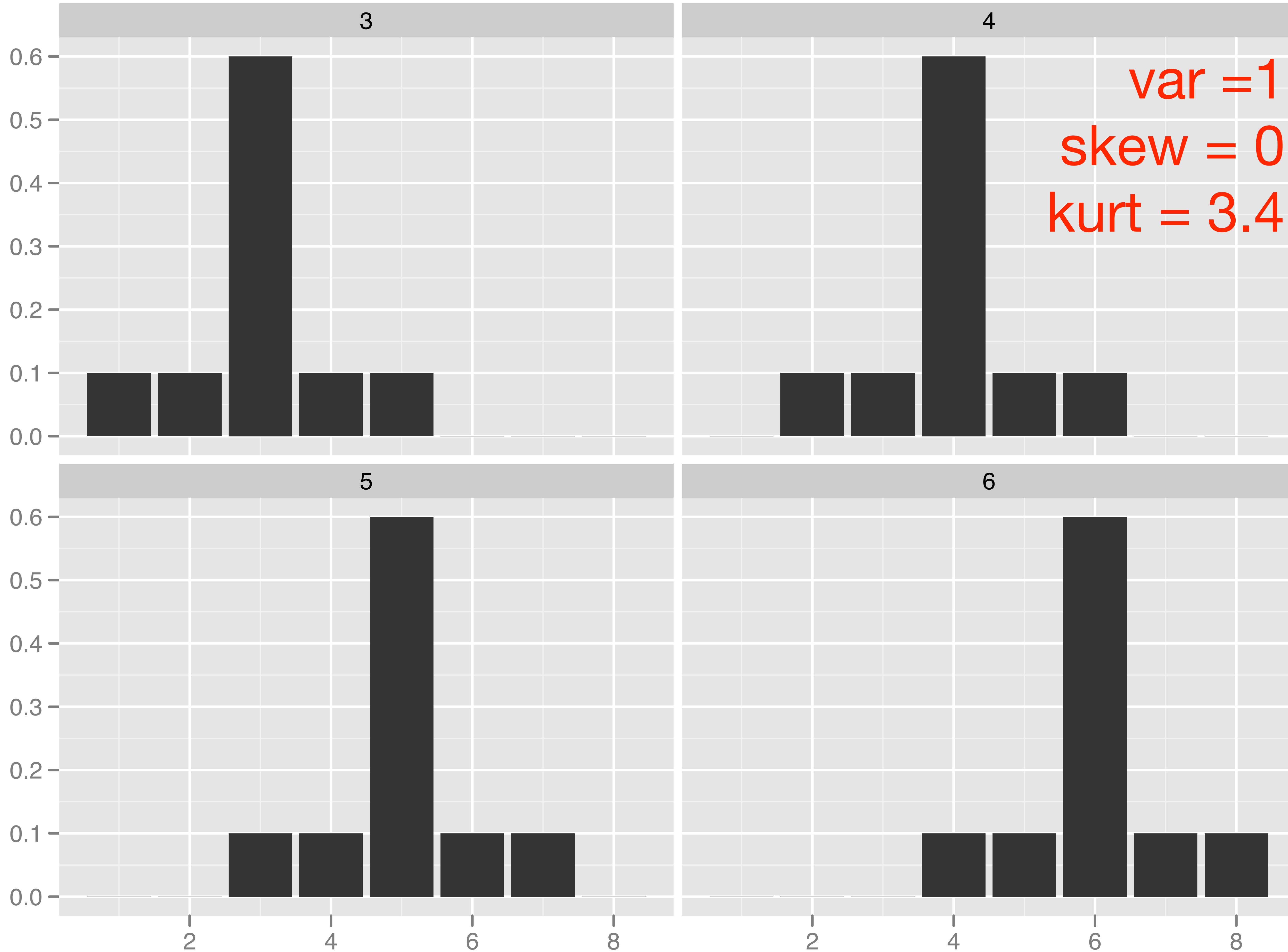
Moments

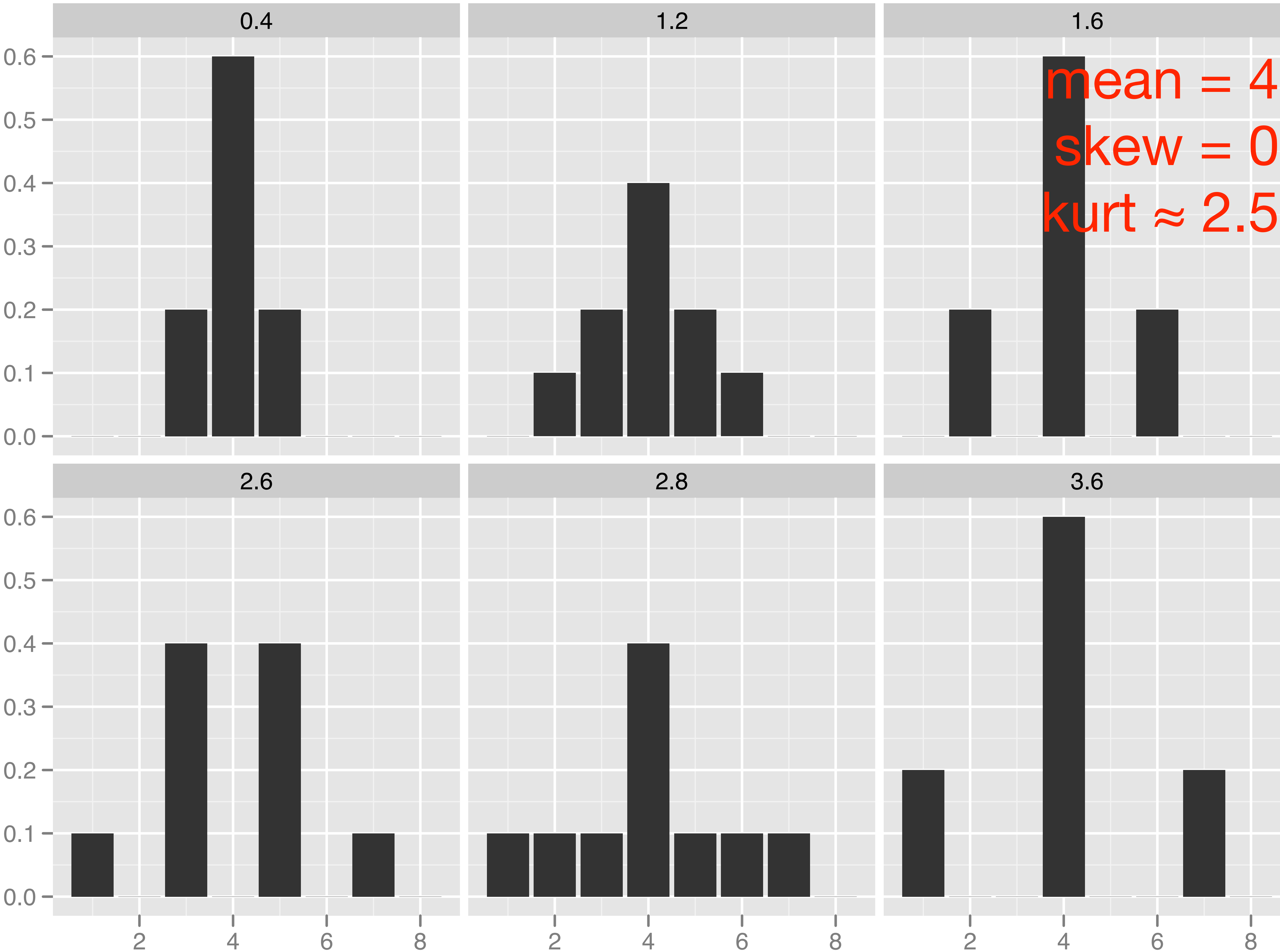
Moments

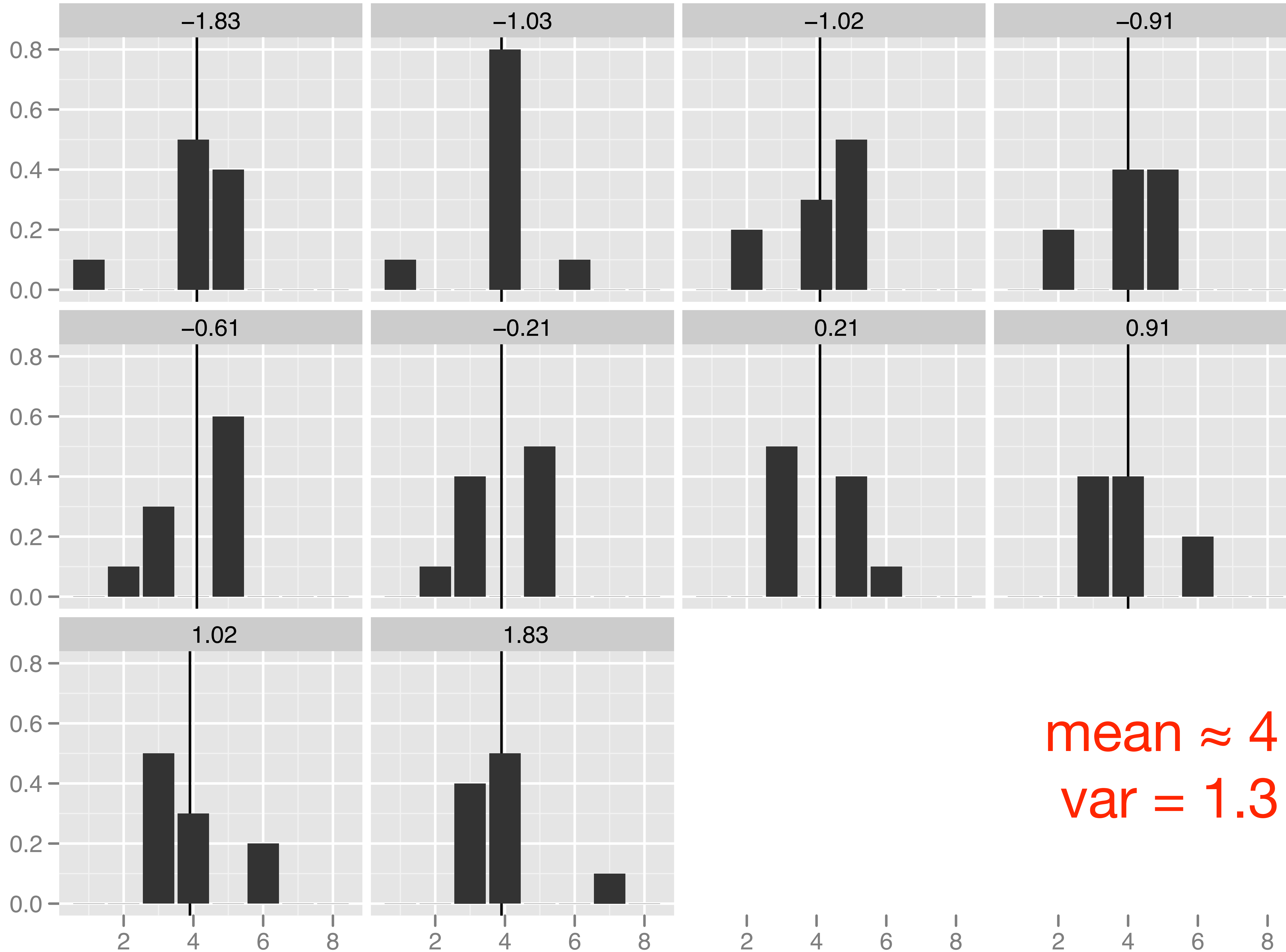
The ith **moment** of a random variable is defined as $E(X^i) = \mu'_i$. The ith **central moment** is defined as $E[(X - E(X))^i] = \mu_i$

The mean is the ________ moment. The variance is the ________ central moment.

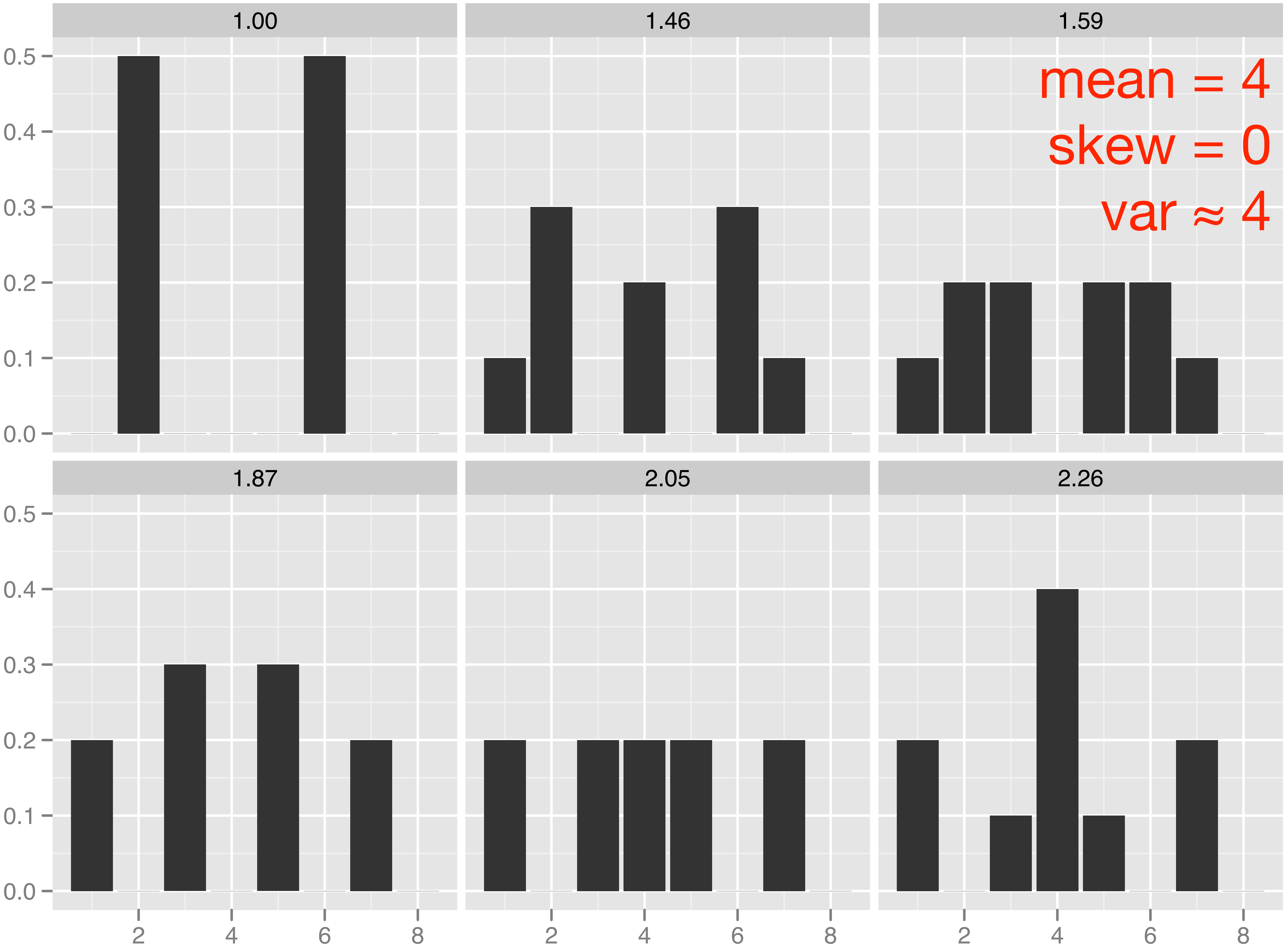
	Name	Symbol	Formula
1	mean	μ	μ'_1
2	variance	σ^2	$\mu_2 = \mu'_2 - \mu^2$
3	skewness	α_3	μ_3 / σ^3
4	kurtosis	α_4	μ_4 / σ^4


3
4
5
6
0.6
0.5
0.4
0.3
0.2
0.1
0.0
2
4
6
8
var =1
skew = 0
kurt = 3.4


0.4
1.2
1.6
2.6
2.8
3.6
mean = 4
skew = 0
kurt ≈ 2.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
2
4
6
8



mean ≈ 4
var = 1.3



1.00
1.46
1.59
1.87
2.05
2.26
mean = 4
skew = 0
var ≈ 4
0.0
0.1
0.2
0.3
0.4
0.5
2
4
6
8

The MGF

mgf

Memorise these three properties

The **moment generating function** (**mgf**) is $M_X(t) = E(e^{Xt})$

(Provided it is finite in a neighbourhood of 0)

Why is it called the mgf?

If $M_X(t) = M_Y(t)$ then X and Y have the same pmf.

Once we have the mgf, it's usually much easier to find the mean and variance

Using the mgf

Expectation of binomial (take 2)

Figure out mgf.
(Random mathematical fact: binomial theorem)

Differentiate & set to zero.

Then work out variance.

Binomial $$(1-p+pe^t)^n$$

Poisson $$\exp(e^{\lambda t}-1)$$

Negative binomial $$\left(\frac{1-p}{1-pe^t}\right)^r$$

Your turn

$$M_X(t) = \exp(e^{\lambda t} - 1)$$

How would you find the mean and variance of the Poisson distribution given the moment generating function?

Wolfram alpha

```
d/dt exp( exp(lambda t) - 1)
d/dt exp( exp(lambda t) - 1) at t = 0
d/dt d/dt exp( exp(lambda t) - 1) at t = 0

d/dt -> differentiate with respect to t
at t = 0 -> evaluate at 0
(show steps can also be useful)
```

Negative binomial

```
choose(x + r - 1, x) (1 - p)^r p^x
sum_(x = 0)^oo choose(x + r - 1, x) (1 -
p)^r p^x
sum_(x = 0)^oo (1- p)^r * p^x * (x+r-1)! /
((r-1)! x!)

sum_(x = 0)^oo x (1- p)^r * p^x * (x
+r-1)! / ((r-1)! x!)
sum_(x = 0)^oo exp(xt) (1- p)^r * p^x * (x
+r-1)! / ((r-1)! x!)
```

Negative binomial

```
d/dt ((1 - p) / (1 - p e^t)) ^ r
d/dt ((1 - p) / (1 - p e^t)) ^ r at t = 0

d^2/dt^2 ((1 - p) / (1 - p e^t)) ^ r at t
= 0
```

Summary

For a new distribution:

Verify that it's a pmf

Compute expected value and variance from definition (given partial proof to complete)

Compute the mgf (given random mathematical fact)

Compute the mean and variance from the mgf (remembering variance isn't second moment)

Recognise and recall

Discrete uniform, Bernoulli, binomial, negative binomial, poisson

Assumptions

Special properties
(e.g. adding binomials with same p, multiplying poissons)

Mean, variance, mgf

Solve problems

Convert word problem in to statistics problem.

Decide which distribution is most appropriate, identify parameters, and justify choice.

Calculate probabilities, means and variances