NEGATIVE BINOMIAL

1

How many successes until r failures

$$\underset{\text{misses}}{\text{successes}} \quad \underset{10}{r} \quad \underset{\text{makes shot}}{\text{failures}}$$

$S^+ = \{\text{number of shots made}\} = 10$

$S^- = \{\text{number of shots missed}\}$

$A = \{\text{attempts}\} \left[= S^+ + S^-\right]$

$S^- \sim \text{Neg Bin}(r = 10,\ p = 1 - 0.86 = 0.14)$

$$E(A) = 10 + E(S^-)$$
$$= 10 + 10\,\frac{0.14}{0.86} = 10 + 1.62$$
$$\approx 12$$

$$P(A > 12) = P(S^- > 2)$$

more than = >

$$= 1 - P(S^- \leq 2)$$
$$= 1 - \left[P(S^- = 2) + P(S^- = 1) + P(S^- = 0)\right]$$
$$= 1 - \left[\binom{10+2-1}{2} 0.86^{10}\, 0.14^2 + \binom{10+1-1}{1} 0.86^{10}\, 0.14 + \binom{10+0-1}{0} 0.86\ 0.14\right]$$

55

10

1

2

$$= 1 - [0.23 + 0.31 + 0.22]$$
$$= 1 - 0.76 = 0.24 \quad \square$$

MOMENTS

$E(X^i) = \mu_i'$ i^{th} ~~central~~ moment

$E((X-\mu)^i) = \mu_i$ i^{th} central moment

$$E(X) = \underset{\mu}{\mu} = \mu_1'$$

$$Var(X) = \mu_2 = \mu_2' - (\mu_1')^2$$

MGF

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

$$E(e^{Xt}) = E\left(1 + Xt + \frac{(Xt)^2}{2!} + \frac{(Xt)^3}{3!} + \dots\right) = E\left(\sum_{i=0}^{\infty} \frac{(Xt)^i}{i!}\right)$$

$$= 1 + tE(X) + \frac{t^2 E(X^2)}{2!} + \frac{t^3 E(X^3)}{3!} + \dots = \sum_{i=0}^{\infty} \frac{t^i}{i!} E(X^i)$$

$M_X(t)$ is a function of two parameters X & t.

3

$$\frac{d}{dt} M_X(t) = E(X) + t\,E(X^2) + \frac{t^2}{2!} E(X^3) + \cdots$$

$$= \sum_{i=\cancel{0}\,1}^{\infty} \frac{i\,t^{i-1}}{i!} E(X^i)$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} E(X^{j+1}) \qquad (j = i-1)$$

$$\left.\frac{d}{dt} M_X(t)\right|_{t=0} = E(X) \qquad !!$$

$$\left.\frac{d^2}{dt^2} M_X(t)\right|_{t=0} = E(X^2)$$