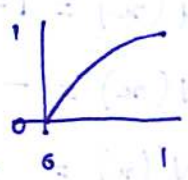
$X \sim \text{Unif}(0,1) \Rightarrow f_X(x) = 1 \quad x \in [0,1]$

$Y = X^{0.5} \Rightarrow f_Y(y) = ? \quad y \in [0,1]$



$$F_Y(y) = P(Y \le y)$$
$$= P(\sqrt{X} \le y)$$
$$= P(X \le y^2)$$
$$= \int_0^{y^2} 1 \, dx$$
$$= [x]_0^{y^2} = y^2 \quad \square$$

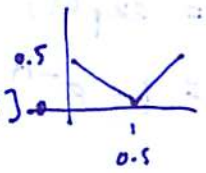
$$F_Y(y) = \begin{cases} 0 & x < 0 \\ y^2 & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} y^2 = 2y \qquad y \in [0,1]$$

(is this a valid pdf?)

$X \sim \text{Unif}(0,1)$

$Z = |X - 0.5| \qquad f_Z(z) = ?$



$\therefore z \in [0, 0.5]$

$$F_Z(z) = P(Z \le z)$$
$$= P(|X - 0.5| \le z)$$
$$-z \le X - 0.5 \le z$$
$$-z + 0.5 \le X \le z + 0.5$$
$$= P(-z + 0.5 \le X \le z + 0.5)$$
$$= \int_{-z+0.5}^{z+0.5} 1 \, dx$$
$$= (z + 0.5) - (-z + 0.5)$$
$$= 2z \qquad z \in [0, 0.5)$$

$f_Z(z) = 1 \quad z > 0.5$

$f_Z(z) = 0 \quad z < 0$

$$f_Z(z) = \frac{d}{dz} 2z = 2 \qquad z \in [0, 0.5]$$

$A = exp(1)$ $\quad S_A = [0, \infty]$

$B = 1 - e^{-A}$

$\lim_{a \to \infty} 1 - B = 0$

$F_B(b) = P(B \le b)$

$= P(1 - e^{-A} \le b)$

$-e^{-A} \le b - 1$

$e^{-A} \ge 1 - b$

$-A \ge \ln(1-b)$

$A \le -\ln(1-b)$

$= P(A \le -\ln(1-b))$

$= F_A(-\ln(1-b))$

$= 1 - e^{-(-\ln(1-b))/1}$

$= 1 - (1-b)$ $\quad b \in [0,1]$

$= b$

$f_B(b) = 1 \qquad b \in [0,1]$

$= B \sim Unif(0,1)$

$u(x) = 1 - e^{-x}$

$v(x) = -\ln(1-x)$

$v'(x) = \frac{-1}{1-x}$

$f_Y(y) = e^{-(-\ln(1-y))} \left| \frac{1}{1-y} \right|$

$= \frac{1-y}{1-y} = 1 \qquad y \in [0,1]$