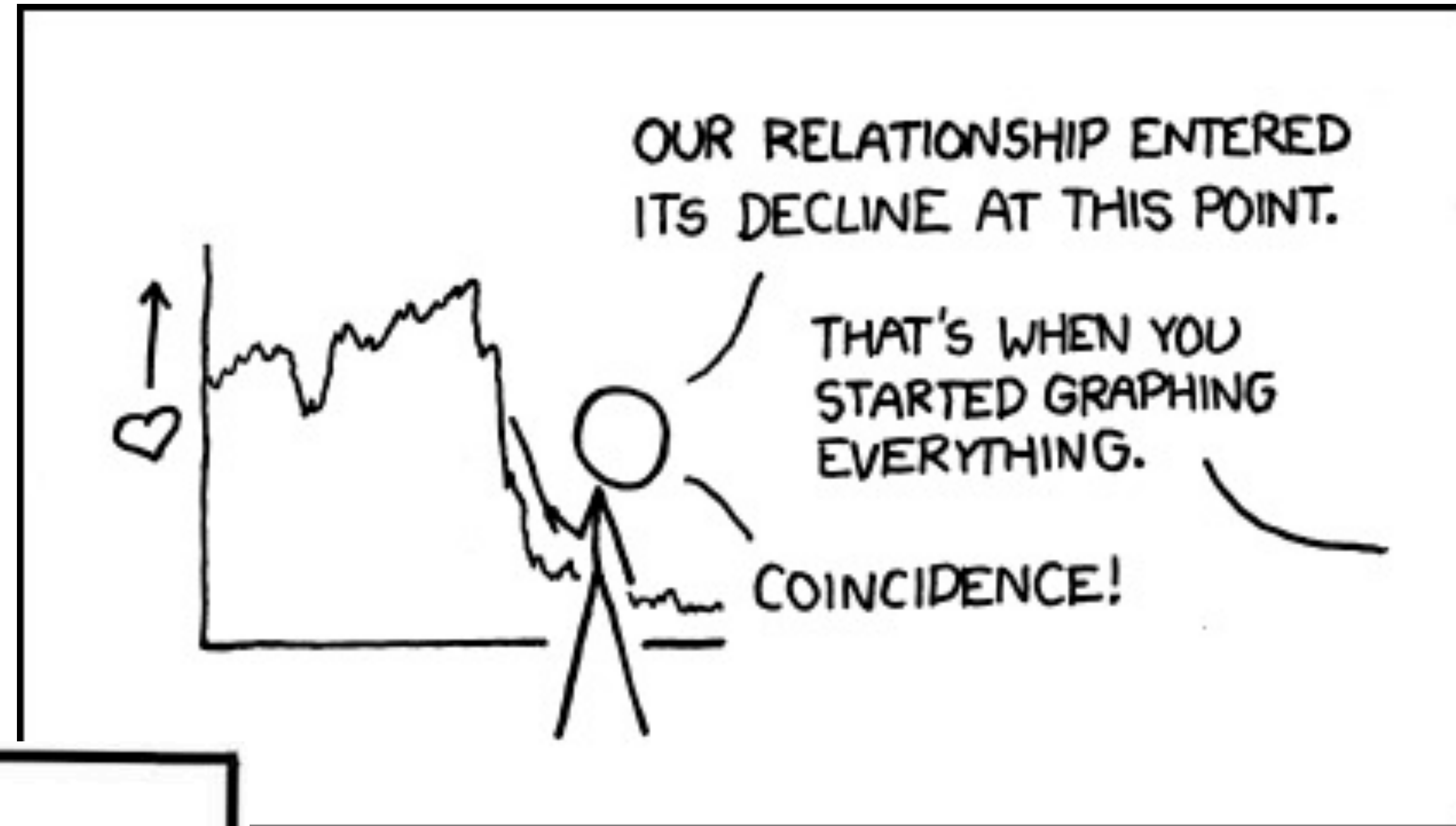
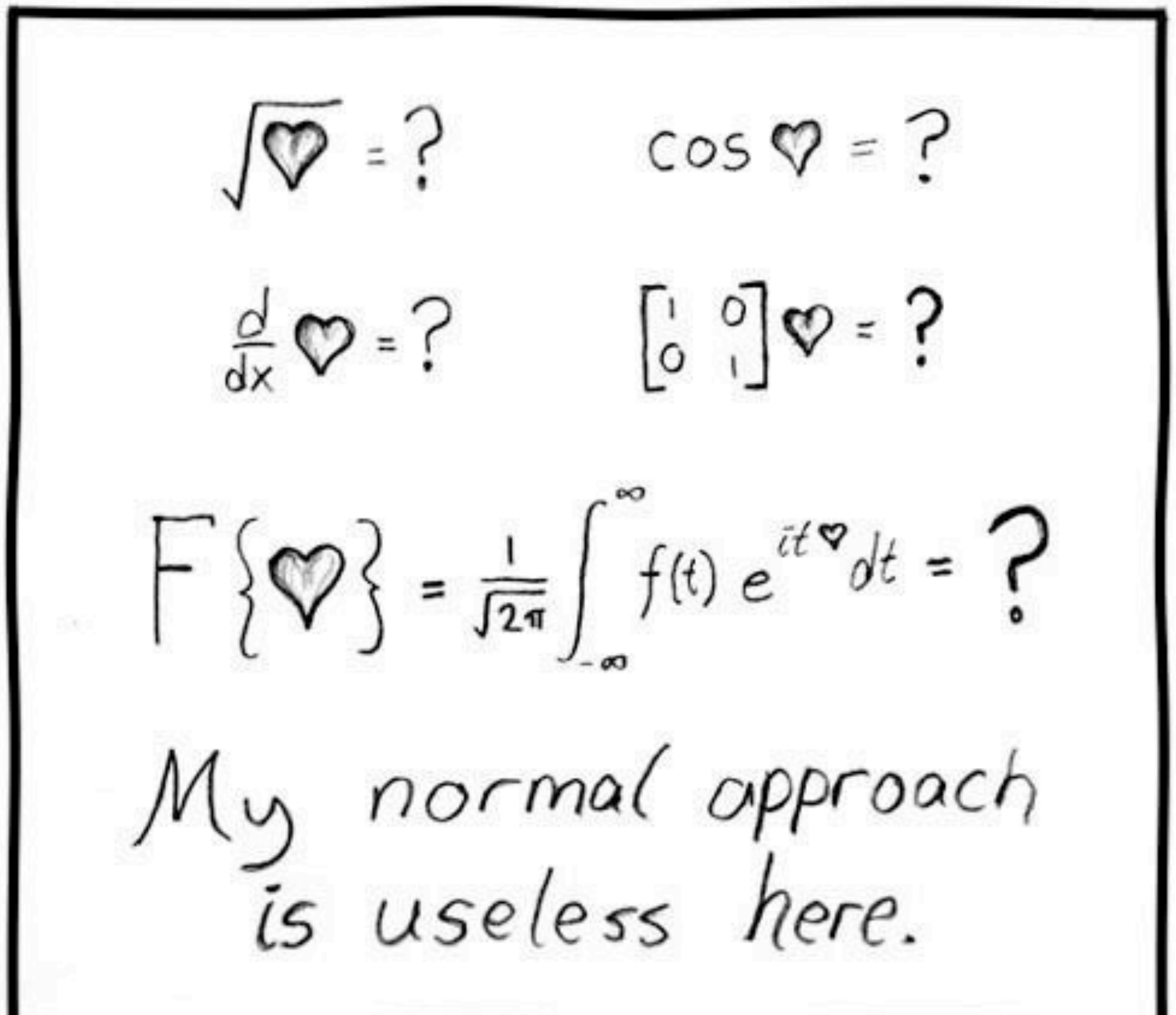



http://xkcd.com/

**Rice Math Tournament**

Saturday, Feb. 18

Herzstein Hall Amphitheater

8:30 AM (breakfast incl.)

1:00 PM (lunch incl.)

rice.eulers@gmail.com

# Stat310

## Review

Hadley Wickham

1. Probability
2. Random variables
3. Transformations

# Probability

Ask questions if anything is unclear!

# Random experiment

"A **random experiment** is an experiment, trial, or observation that can be repeated numerous times under the same conditions... It must in no way be affected by any previous outcome and cannot be predicted with certainty." (http://cnx.org/content/m13470/latest/)

i.e. it is **uncertain** (we don't know ahead of time what the answer will be) and **repeatable** (ideally).

# Sample space

The **sample space** is the **set** containing all possible **outcomes** from a random experiment.  Often called S.

In set theory called **U**
For rv's, often called the **support**

# More terminology

An **event** is a ...

A collection of events are **mutually exclusive** if...

A collection of events are **exhaustive** if...

A collection of events is a **partition** if ...

| Regular | Conditional | |
|---|---|---|
| $P(A) \geq 0$ | $P(A \mid C) \geq 0$, | for all $A \subset S$ |
| $P(S) = 1$ | $P(S \mid C) = 1$ | |
| $P(A \cup B) = P(A) + P(B)$ | $P(A \cup B \mid C) = P(A \mid C) + P(B \mid C)$ | if $A \cap B = 0$ |

| Regular | Conditional |
| --- | --- |
| $P(A') = 1 - P(A)$ | $P(A' \mid C) = 1 - P(A \mid C)$ |
| $P(A) \leq P(B)$ if $A \subset B$ | $P(A \mid C) \leq P(B \mid C)$ if $A \subset B$ |
| $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ | $P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$ |

# Independence

If A and B are **independent**, then
$P(A \cap B) = \ldots$

This implies $P(A \mid B) = \ldots$ ?

Events are **mutually independent** if ...

# Your turn

With your neighbour, recall the **six** tools in your probability toolbox

# Toolbox

Complements

Convert union to sum

Convert intersection to conditioning
(and vice versa)

Convert intersection to product
(if independent)

Use law of total probability

Switch conditioning (Bayes' rule)

# Consequence

Let W be the waiting time in minutes until a (very unreliable) light bulb fails. W ~ Exponential($\theta = 30$).

What's the probability that the light bulb lasts at least 30 minutes?

What's the probability that the light bulb lasts at least 60 minutes given that it's lasted at least 30 minutes?

# Tower of abstraction

$W \sim \text{Exponential}(\theta = 30)$
$P(W > 60 \mid W > 30)$

$W \sim \text{Exponential}(\theta)$
$P(W > 60 \mid W > 30)$

$W \sim \text{Exponential}(\theta)$
$P(W > x + h \mid W > x)$

# Counting

- Multiplication principle
- If order doesn't matter, divide by total possibilities by number of ways of reordering them

# Random variables

# Definitions

A **random variable** is a **random experiment** with a numeric sample space.

A **discrete** random variable has finite or countably infinite sample space. Subset of the integers. Use **sums**. Has p**m**f.

A **continuous** random variable has uncountably infinite sample space. Subset of the real line. Use **integrals**. Has p**d**f.

# Why densities?

Can approximate any continuous distribution as a discrete distribution. (**Demo**)

Density arises as we think of collecting more and more data, and getting a better and better approximation.

Limit as n goes to infinity.

$$f(x)$$

For continuous x, f(x) is a probability **density** function.

# Not a probability!

(may be greater than one)

$$P(a < X < b) = \int_a^b f(x)dx$$

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

What does P(X = a) = ?

$$P(a < X < b) = \int_a^b f(x)dx$$

To get probability, **always** need to integrate

What does P(X = a) = ?

# Your turn

Write down the conditions for a function being a probability function, a pmf or a pdf.

| Probability | Discrete | Continuous |
|---|---|---|
| P(A) ≥ 0<br>for all A ⊂ S | f(x) > 0<br>for all x ⊂ S | f(x) > 0<br>for all x ⊂ **R** |
| P(S) = 1 | $\sum_{x_i \in S} f(x_i) = 1$ | $\int_{\mathbb{R}} f(x) = 1$ |

$$E(g(X)) = \sum_{x \in S} f(x)g(x)$$

$$E(g(X)) = \int_{\mathbb{R}} g(x)f(x)dx$$

little x vs. big x

$$F(x) = P(X \leq x)$$

discrete

continuous

$$F(x) = P(X \leq x)$$

discrete

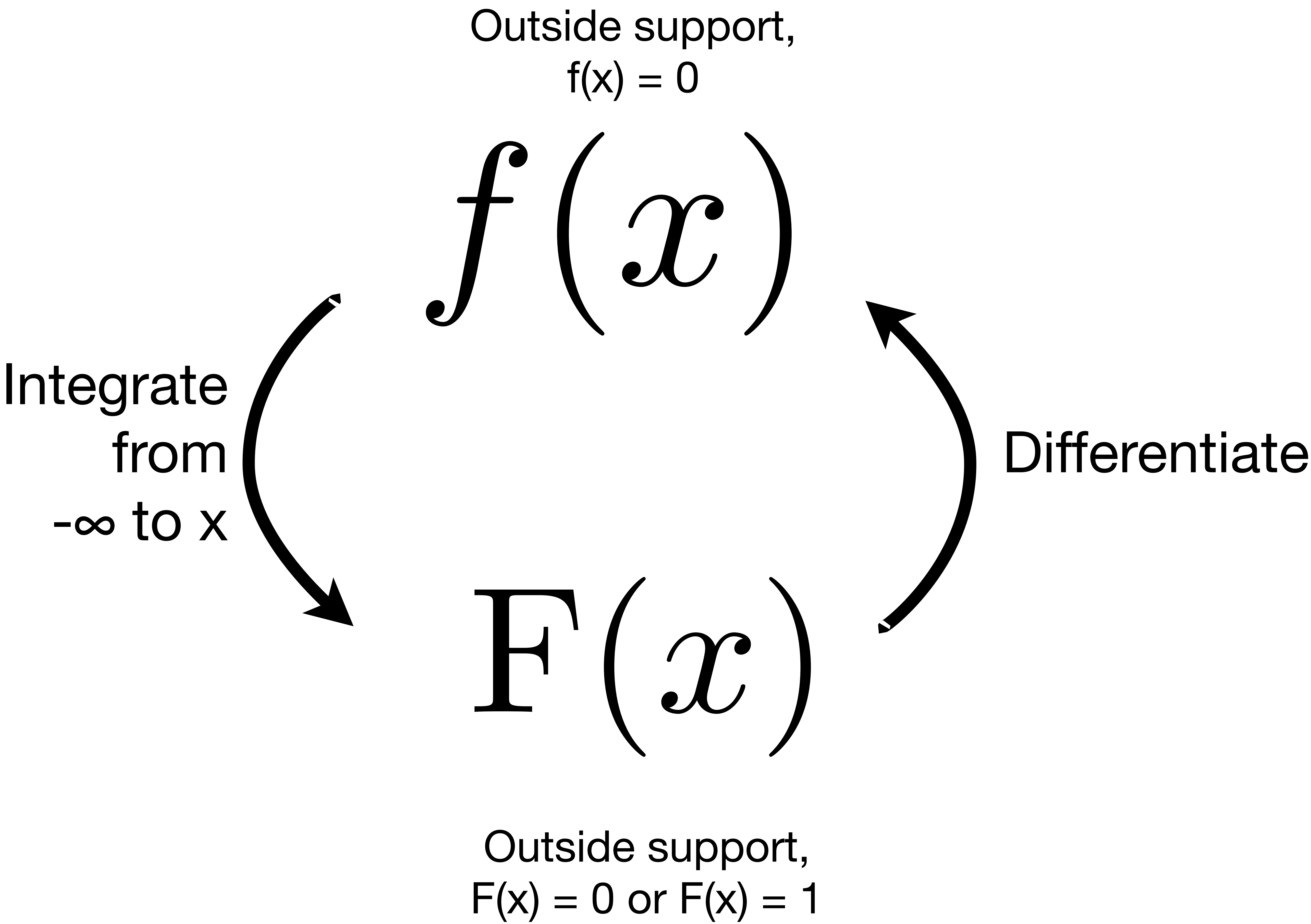
$$F(x) = \sum_{t \leq x} f(t)$$

continuous

$$F(x) = P(X \leq x)$$

discrete $$F(x) = \sum_{t \leq x} f(t)$$

continuous $$F(x) = \int_{-\infty}^{x} f(t)dt$$


Outside support,
f(x) = 0
$f(x)$
Integrate
from
-∞ to x
Differentiate
$\mathrm{F}(x)$
Outside support,
F(x) = 0 or F(x) = 1

# Moments

The ith **moment** of a random variable is defined as $E(X^i) = \mu'_i$. The ith **central moment** is defined as $E[(X - E(X))^i] = \mu_i$

The mean is the ________ moment. The variance is the ________ central moment.

# mgf

Memorise these three properties

The **moment generating function** (**mgf**) is $M_X(t) = E(e^{Xt})$

(Provided it is finite in a neighbourhood of 0)

Why is it called the mgf?

If $M_X(t) = M_Y(t)$ then X and Y have the same pmf.

# Finding moments

Directly from expectation

With moment generating function

# Transformations

# Transformations

Distribution
function
technique

(always works, but hard)

Change of
variable
technique

(easy, but doesn't always work)

Much more useful for 2+ rvs

# CDF technique

Y = u(X)

Work out range.

**Substitute** X into cdf of Y

**Rearrange** to get probability in terms of X.

**Integrate** to find cdf.

**Differentiate** to find pdf.

# Your turn

$A \sim Exp(1)$

$B = 1 - e^{-A}$

Find $f_Y(y)$. Does y have a named distribution?