VARIANCE

$$\mathrm{Var}(aX+b) = E[(aX+b)^2] - (E[aX+b])^2$$

$$= E[a^2X^2 + 2abX + b^2] - (aE[X]+b)^2$$

$$= a^2E[X^2] + \cancel{2ab E[X]} \cancel{+b^2} - a^2E[X]^2 \cancel{- 2abE[X]} \cancel{- b^2}$$

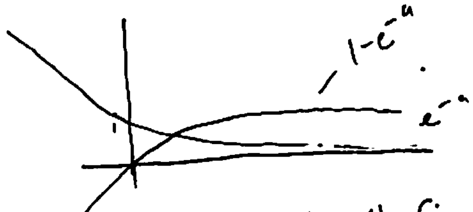
$$= a^2(E[X^2] - E[X]^2)$$

$$= a^2\,\mathrm{Var}(X) \quad \square$$

$A = Exp(1) \Rightarrow f_A(a) = e^{-a} \quad F_A(a) = 1 - e^{-a}$

$a > 0$

$B = 1 - e^{-A}$



$\Rightarrow$ ~~M~~ $b \in [0,1]$

(can't figure out how to do in WA - if you do please let me know!)

$u(x) = 1 - e^{-x} = y$

$1 - y = e^{-x}$

$-x = \ln(1-y)$

$x = -\ln(1-y) \Rightarrow v(x) = -\ln(1-x)$

$v'(x) = \frac{1}{1-x}$

$f_Y(y) = f_X(v(y))\,|v'(y)|$

$f_B(b) = f_A(v(b))\,|v'(b)|$

$= f_A(-\ln(1-b))\,\left|\frac{1}{1-b}\right|$

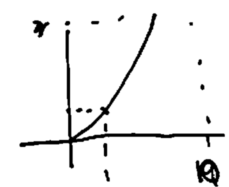
$= e^{-\ln(1-b)}\,\frac{1}{1-b}$

$= \frac{1-b}{1-b} = 1 \quad \square \qquad b \in [0,1]$

$\Rightarrow B \sim \text{Uniform}(0,1)$.

$X = \text{Unif}(1,10) \Rightarrow f_X(x) = \frac{1}{9} \quad x \in [1,10]$

$Y = X^2$



$y \in$ [1,100]

$u(x) = x^2 = y$

$x = \sqrt{y} \Rightarrow v(x) = \sqrt{x}$

$v'(x) = \frac{1}{2\sqrt{x}}$

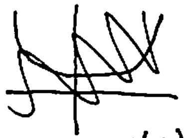
$f_X(x) = f_Y(v(x))\,|v'(x)|$

$= f_Y(\sqrt{x})\,\left|\frac{1}{2\sqrt{x}}\right|$

$= \frac{1}{9 \cdot 2\sqrt{x}} \quad \square \qquad x \in [1,100]$

$f_c(c) = \frac{c^2}{9} \qquad c \in [0,3]$

$D = \frac{1}{1+d^2}$



$u(0) = \frac{1}{1+0} = \frac{1}{2}$

$u(3) = \frac{1}{1+9} = \frac{1}{10}$

$u(x) = \frac{1}{1+d^2} = y$

$d \in [\frac{1}{10}, \frac{1}{2}]$

$1+d^2 = \frac{1}{y}$

$d^2 = \frac{1}{y} - 1$

$\Rightarrow v(x) = \frac{1}{x} - 1 = \frac{1-x}{x}$

$v'(x) = -\frac{1}{x^2}$

$$f_D(d) = f_c(v(d))\,|v'(d)|$$

$$= f_c\left(\frac{1}{d} - 1\right) \left| -\frac{1}{d^2} \right|$$

$$= \frac{\left(\frac{1}{d} - 1\right)^2}{9} \frac{1}{d^2}$$

$$= \frac{(1-d)^2}{9d^4} \qquad d \in [\tfrac{1}{4}, \tfrac{1}{2}]$$

$f_c(c) = \frac{c^2}{9} \qquad c \in [0,3]$

$D = \frac{1}{1+C^2} \qquad d \in \left[\frac{1}{1+3^2}, \frac{1}{1+0^2}\right] = \left[\frac{1}{10}, 1\right]$

$u(x) = \frac{1}{1+x^2} = y$

$x^2+1 = \frac{1}{y}$

$x^2 = \frac{1}{y} - 1$

$x = \sqrt{\frac{1}{y} - 1} \qquad \Rightarrow v(x) = \sqrt{\frac{1}{x} - 1}$

$v'(x) = \frac{-1}{2x^2\sqrt{\frac{1}{x} - 1}}$

$$f_D(d) = f_c(v(d))\,|v'(d)|$$

$$= f_c\left(\sqrt{\frac{1}{d} - 1}\right) \left| \frac{-1}{2d^2\sqrt{\frac{1}{d} - 1}} \right|$$

$$= \frac{\frac{1}{d} - 1}{9} \cdot \frac{1}{2d^2\sqrt{\frac{1}{d} - 1}}$$

$f(x) \longrightarrow f(x,y)$

$f(x,y) = x^2 + yx$

$\frac{\partial}{\partial x} f(x,y) = 2x + y$ ← treat y as constant differentiate wrt x

$\frac{\partial}{\partial y} f(x,y) = x^2 + x$ ← treat x as constant different wrt x

$\frac{\partial^2}{\partial x \partial y} f = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f(x,y)\right) = 2x + 1$

$= \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f(x,y)\right) = 2x + 1$

↑ order doesn't matter.

$\iint f(x,y)\,dx\Big)\,dy = \int\left(\int f(x,y)\,dy\right)\,dx$

$\int f(x,y)\,dy = x^2 y + \frac{xy^2}{2} + c$

$\int f(x,y)\,dx = \frac{x^3}{3} + \frac{yx^2}{2} + c$

$\int_{\mathbb{R}}\int_{\mathbb{R}} = \iint_{\mathbb{R}^2} = \iint_S$ for pdfs (why?)

Find volume if height $= x^2 + yx$ over $0 < x, y < 1$ ← $0 < x < 1$, $0 < y < 1$

$(x,y) \in [0,1] \times [0,1]$

$x \in [0,1],\ y \in [0,1]$

$$\int_0^1\left(\int_0^1 x^2 + yx \, dx\right) dy$$

$$= \int_0^1 \left[\frac{x^3}{3} + \frac{x^2 y}{2}\right]_0^1 dy$$

$$= \int_0^1 \frac{1}{3} + \frac{y}{2}\, dy$$

$$= \left[\frac{1}{3}y + \frac{y^2}{4}\right]_0^1$$

$$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}.$$

Integrate x^2 + yx from x = 0 to 1, y = 0 to 1