Stat310

Transformations / Bivariate

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1. Statistics in practice
2. Clarifications
3. Transformations
4. Intro to bivariate distributions

Statistics in practice

Stats in practice

- Due Thursday March 8
- Read an article about statistics in real life and write a one page response.
- Why did you pick the article? What did you learn and how does it connect to stat310? What questions are you left with?
- **Do not** summarise the article

Clarifications

Variance

$$Var(aX + b) = a^2 \, Var(X)$$

Transformations

If $X \sim \text{Normal}(\mu, \sigma^2)$, and $Y = aX + b$

THEN $Y \sim \text{Normal}(a\mu + b, a^2\sigma^2)$

Properties of expectation tell us what the mean and variance must be, but don't tell us what distribution it has.

Transformations

Transformations

Distribution function technique

(always works, but hard)

Change of variable technique

(easy, but doesn't always work)

Change of variables

Transformation must have an inverse!

If $Y = u(X)$, and

v is the inverse of u, $X = v(Y)$

then

$$f_Y(y) = f_X(v(y))\,|v'(y)|$$

Practice

$A \sim \text{Exp}(1)$

$B = 1 - e^{-A}$

Find $f_Y(y)$. Does y have a named distribution?

Your turn

$X = \text{Uniform}(1, 10)$

$Y = X^2$

More practice

C has pdf $f(c) = c^2/9 \quad 0 < c < 3$

$D = 1 / (1 + C^2)$

Relationship to uniform

Important connection between the uniform and every other random variable through the cdf.

Uniform to any rv

IF

$Y \sim$ Uniform(0, 1)

F a cdf

THEN

$X = F^{-1}(Y)$ is a rv with cdf $F(x)$

(Assume F strictly increasing for simplicity)

Any rv to uniform

IF

X has cdf F

Y = F(X)

THEN

Y ~ Uniform(0, 1)

(Assume F strictly increasing for simplicity)

Bivariate random variables

Bivariate rv

A **random experiment** where we measure **two things** (not just one). A vector instead of a single observation.

These variables could be both discrete, both continuous, or one continuous and one discrete. We will focus on both continuous: a **bivariate continuous random variable**.

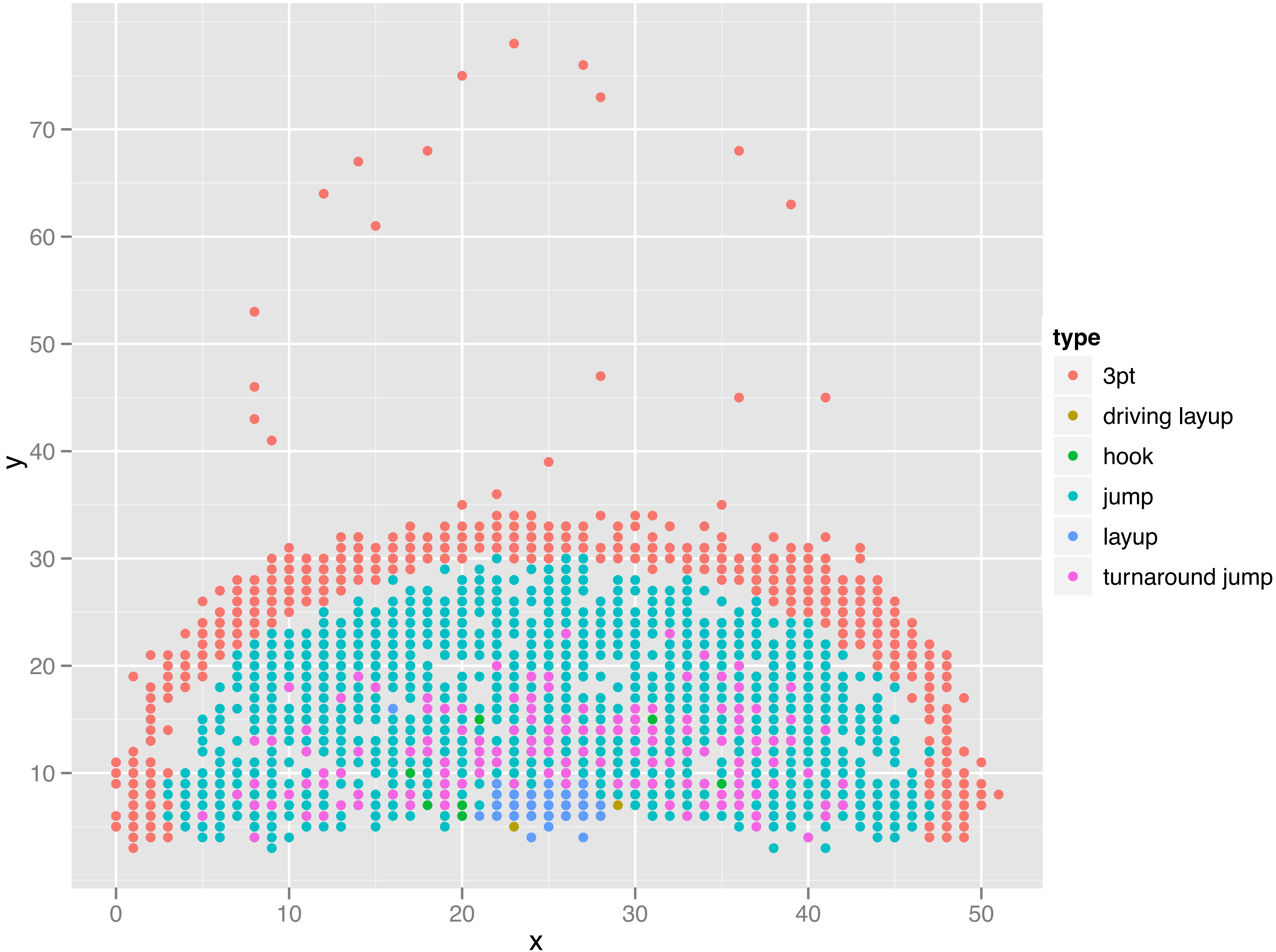
Sample space

Univariate continuous rv: sample space is an interval on the real line.

Bivariate continuous rv: sample space is region on the real plane.

$$S = \{(x, y) : f(x, y) > 0\}$$

Example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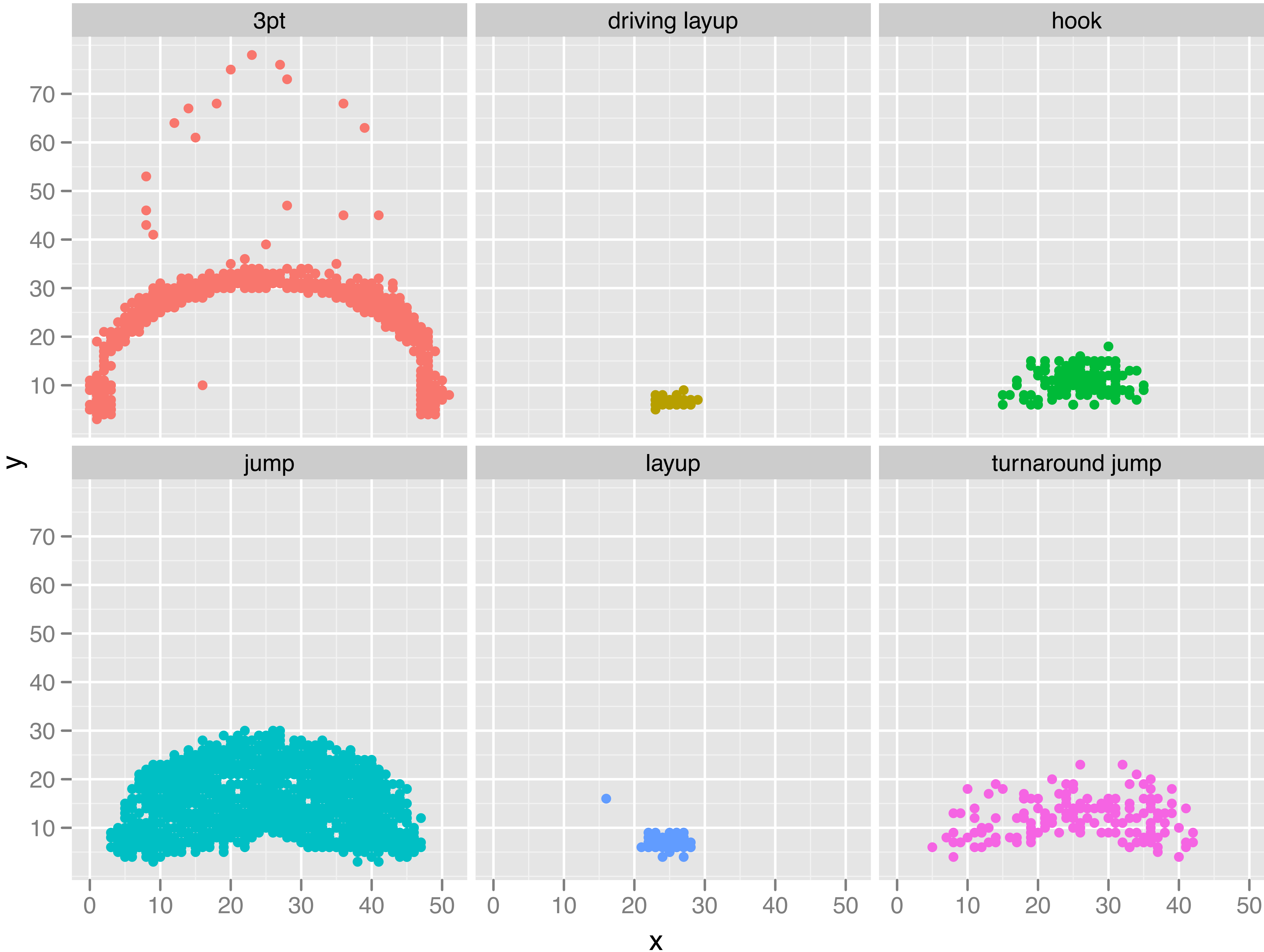

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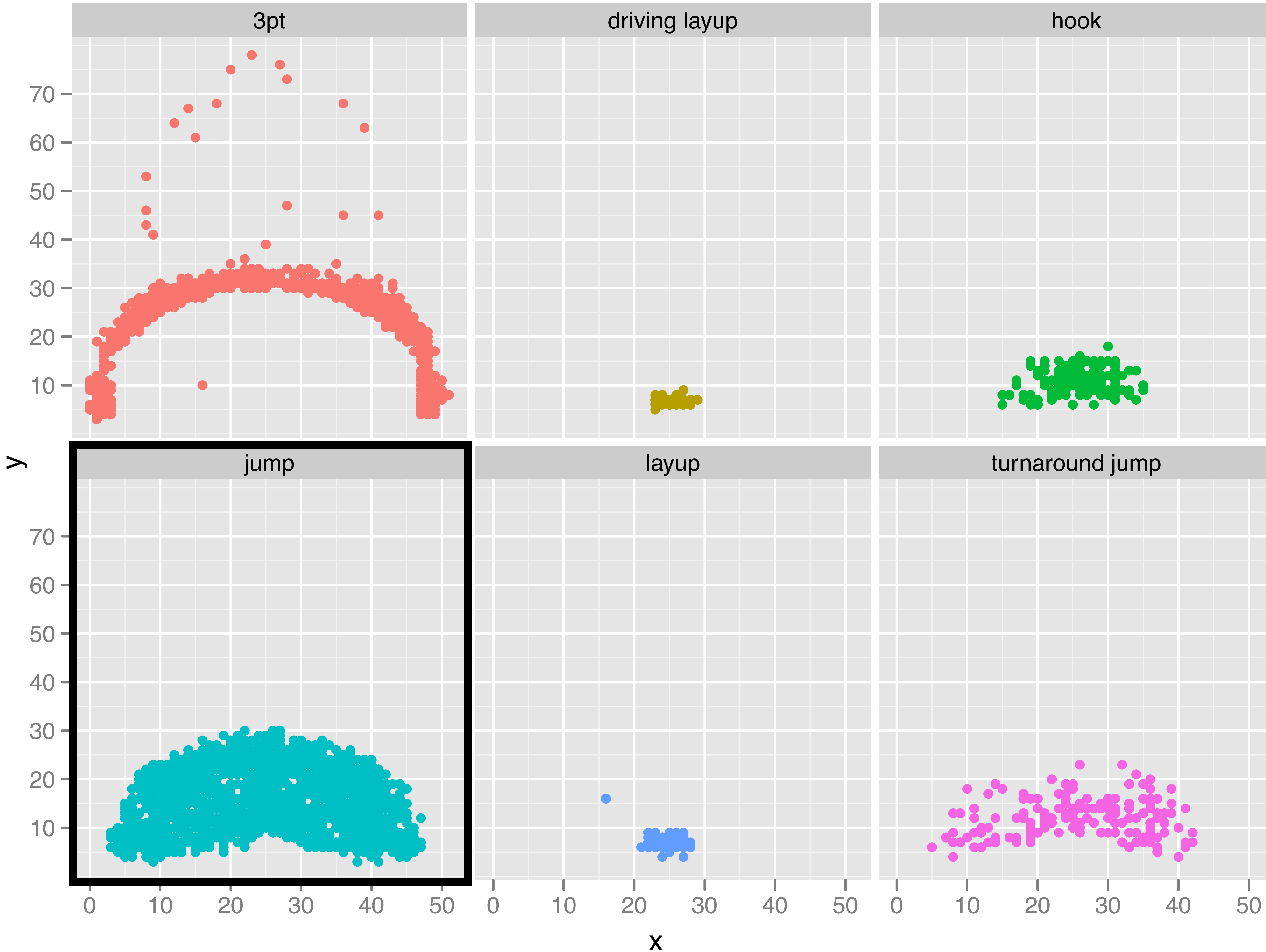
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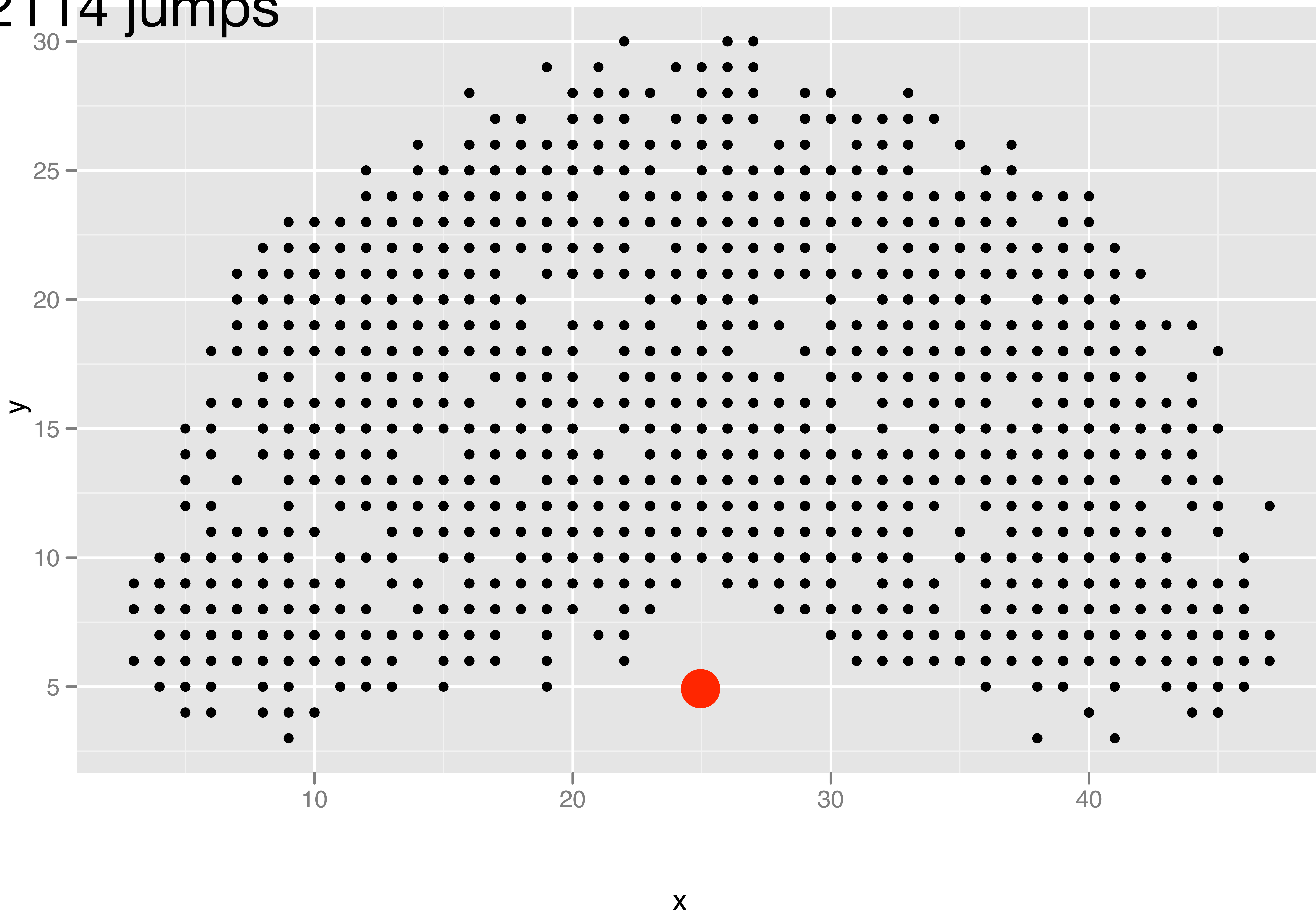
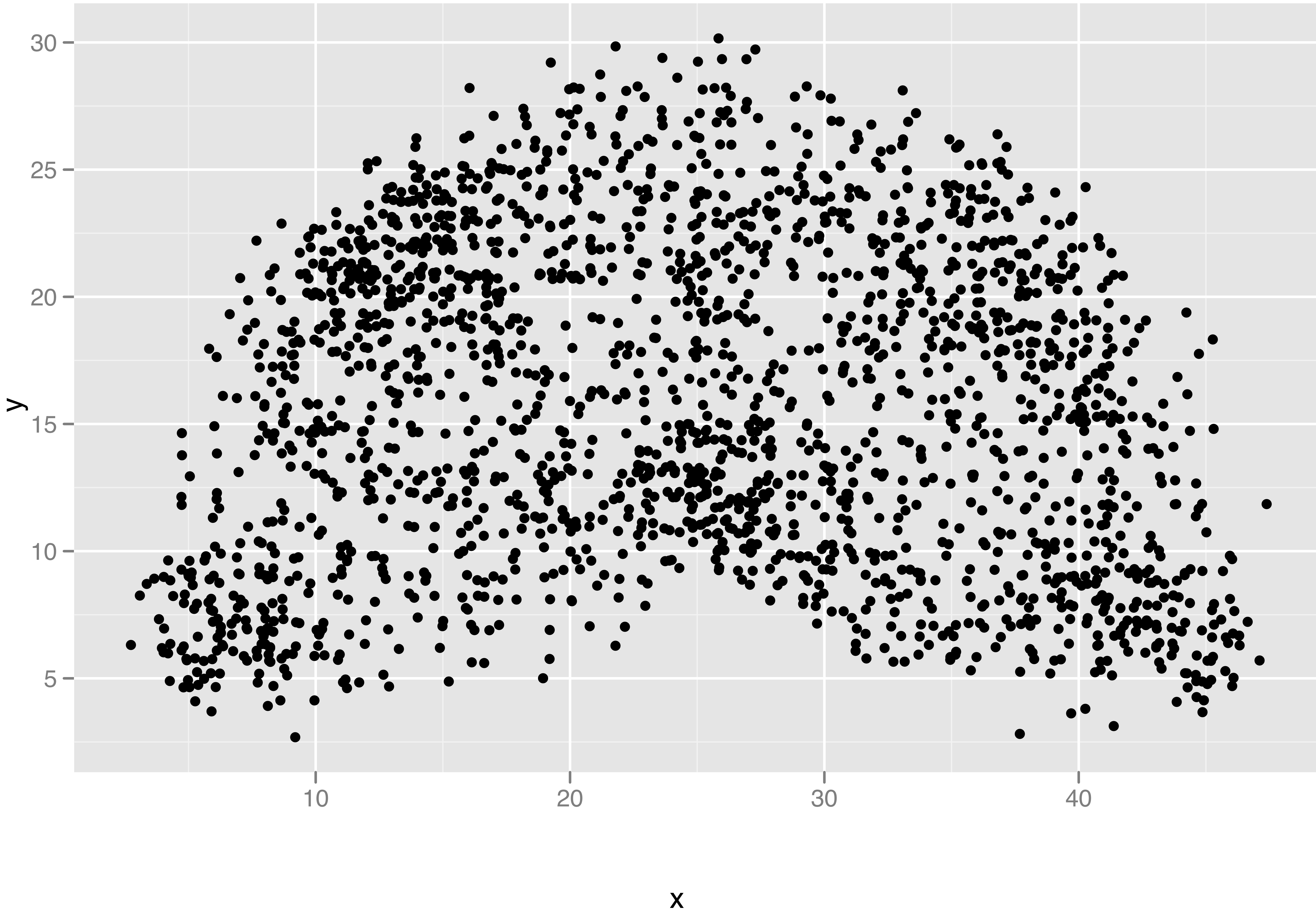
Joint distribution of x and y

2114 jumps

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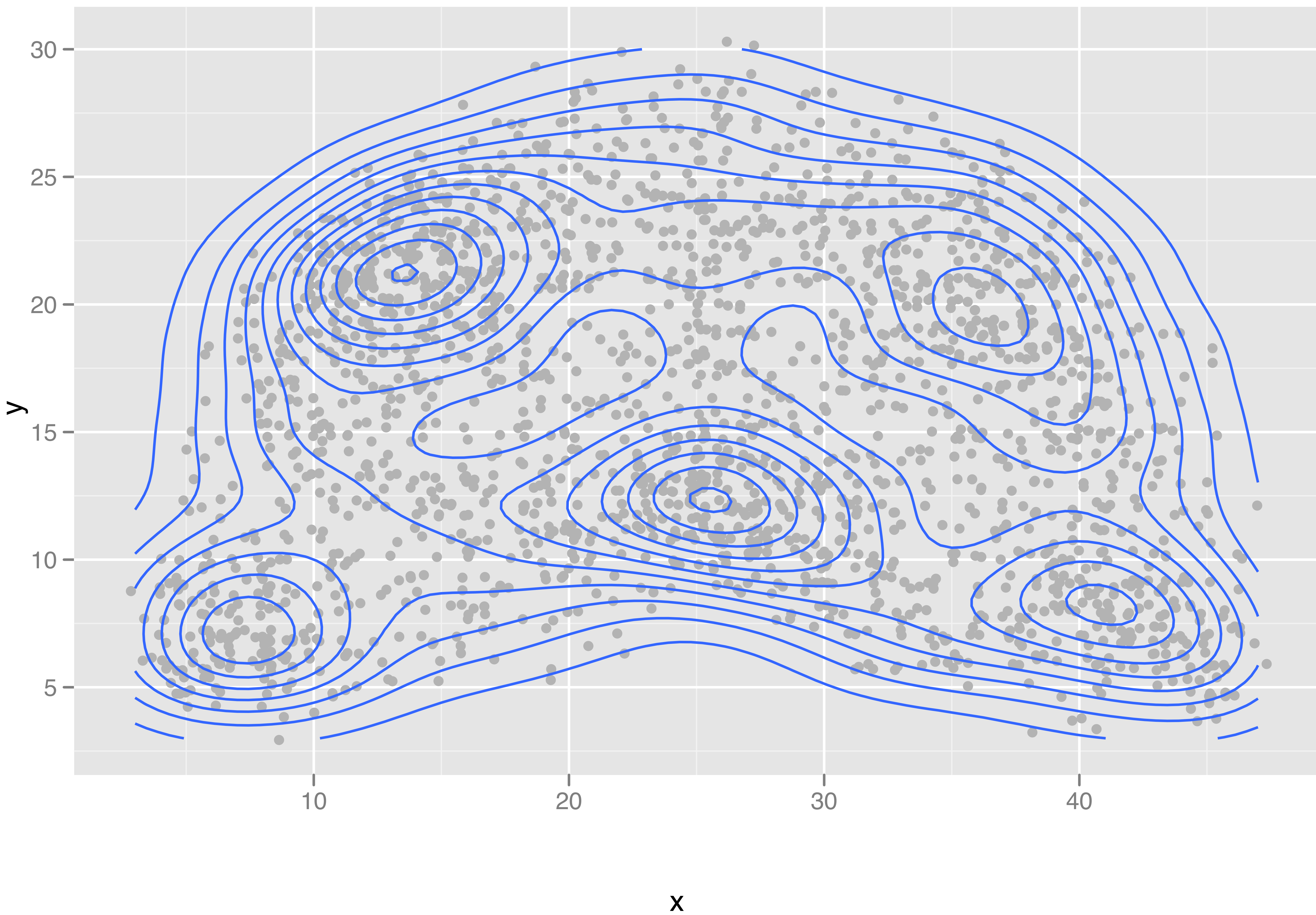
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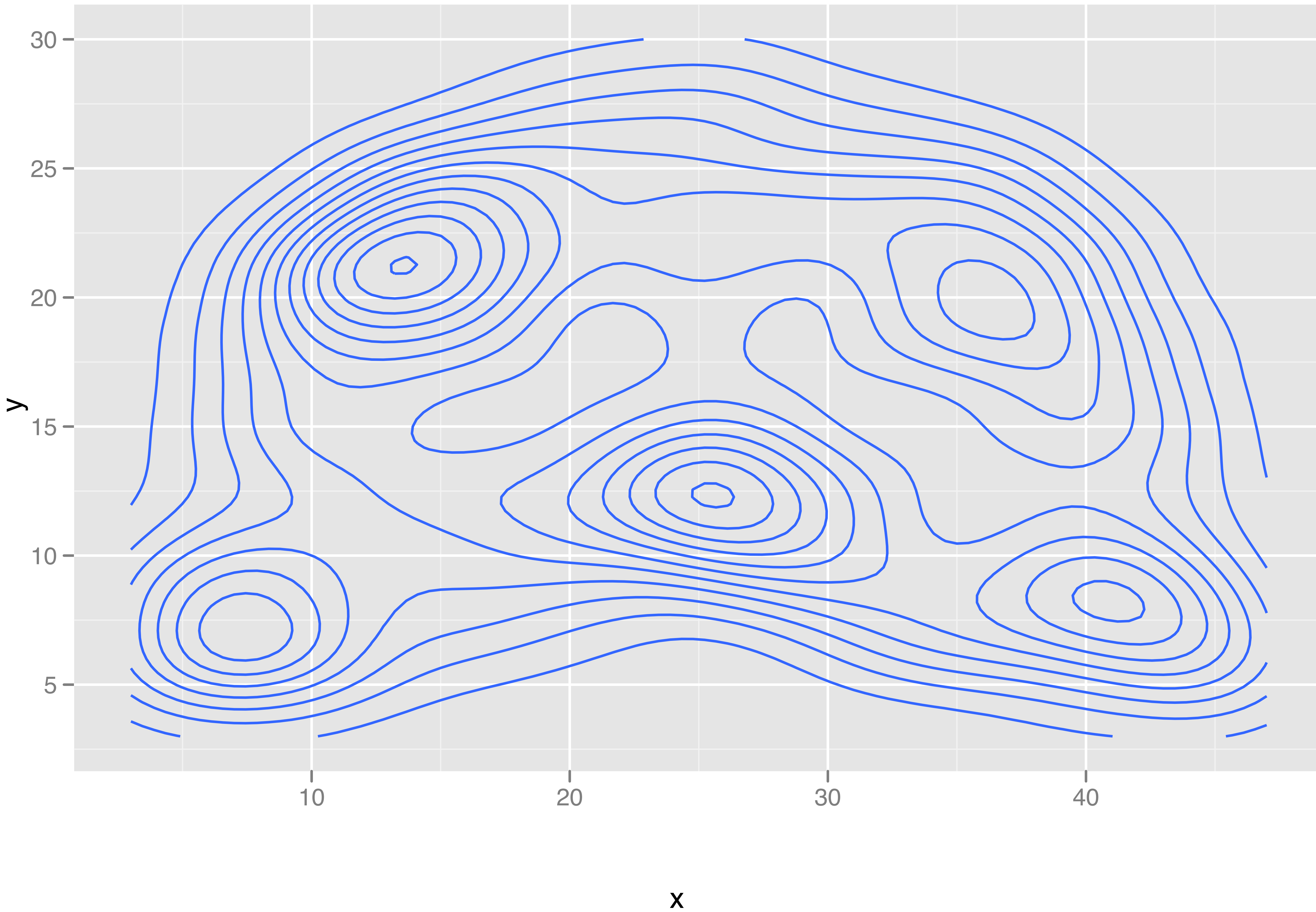

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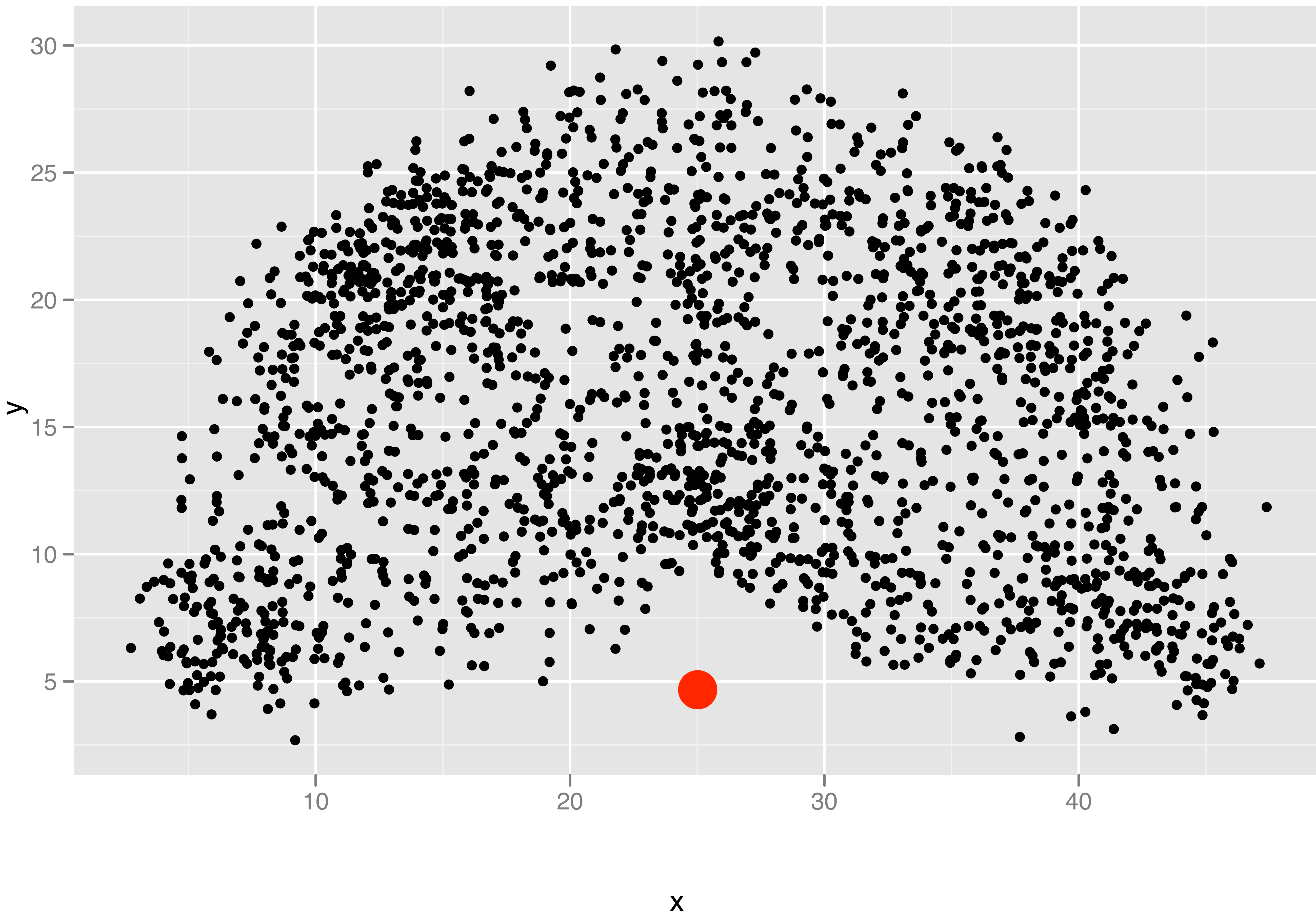
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Your turn

Take a minute to sketch out what you think the distributions of x and y will look like.

2d

1d

Marginal

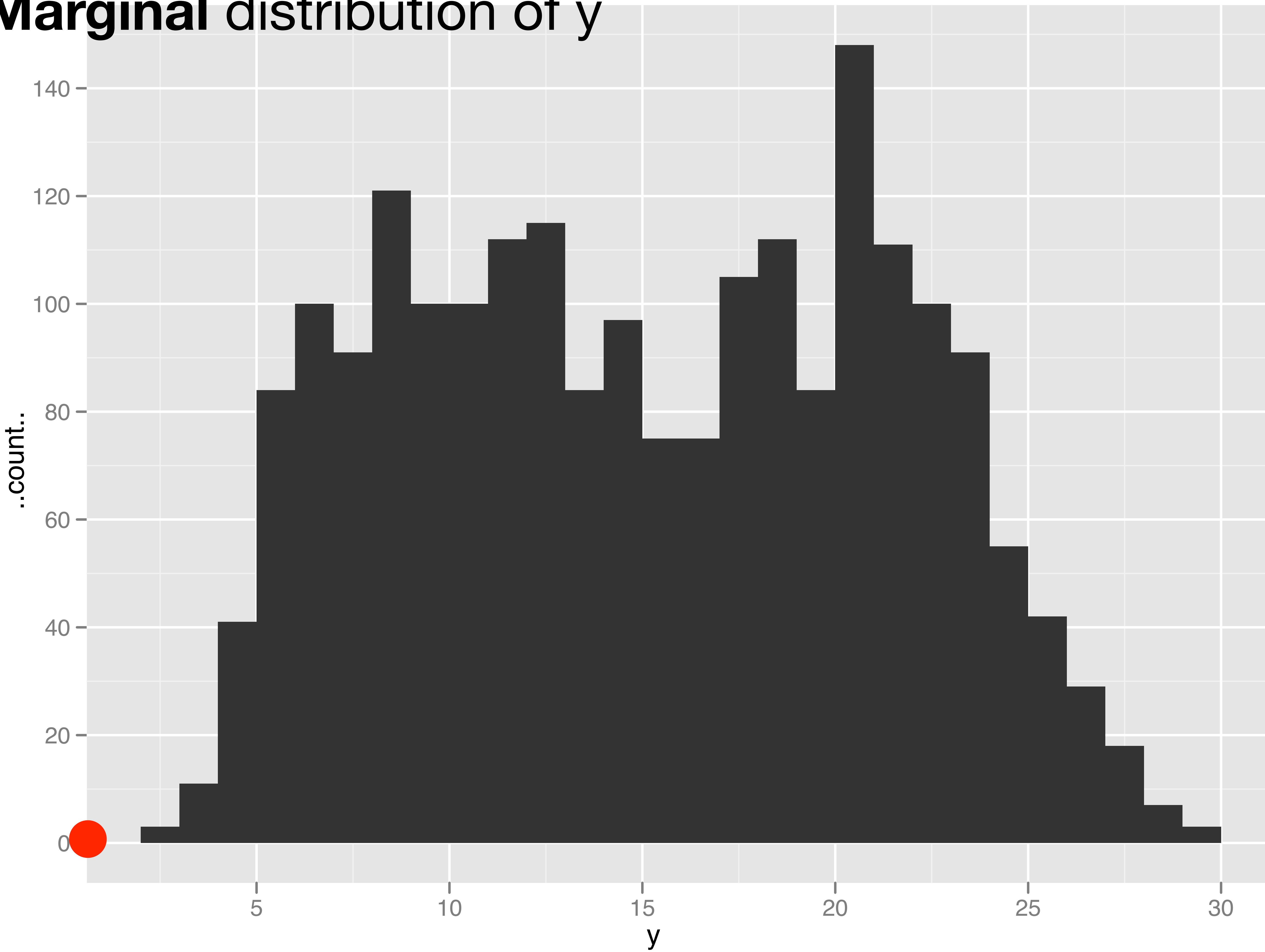
integration

Joint

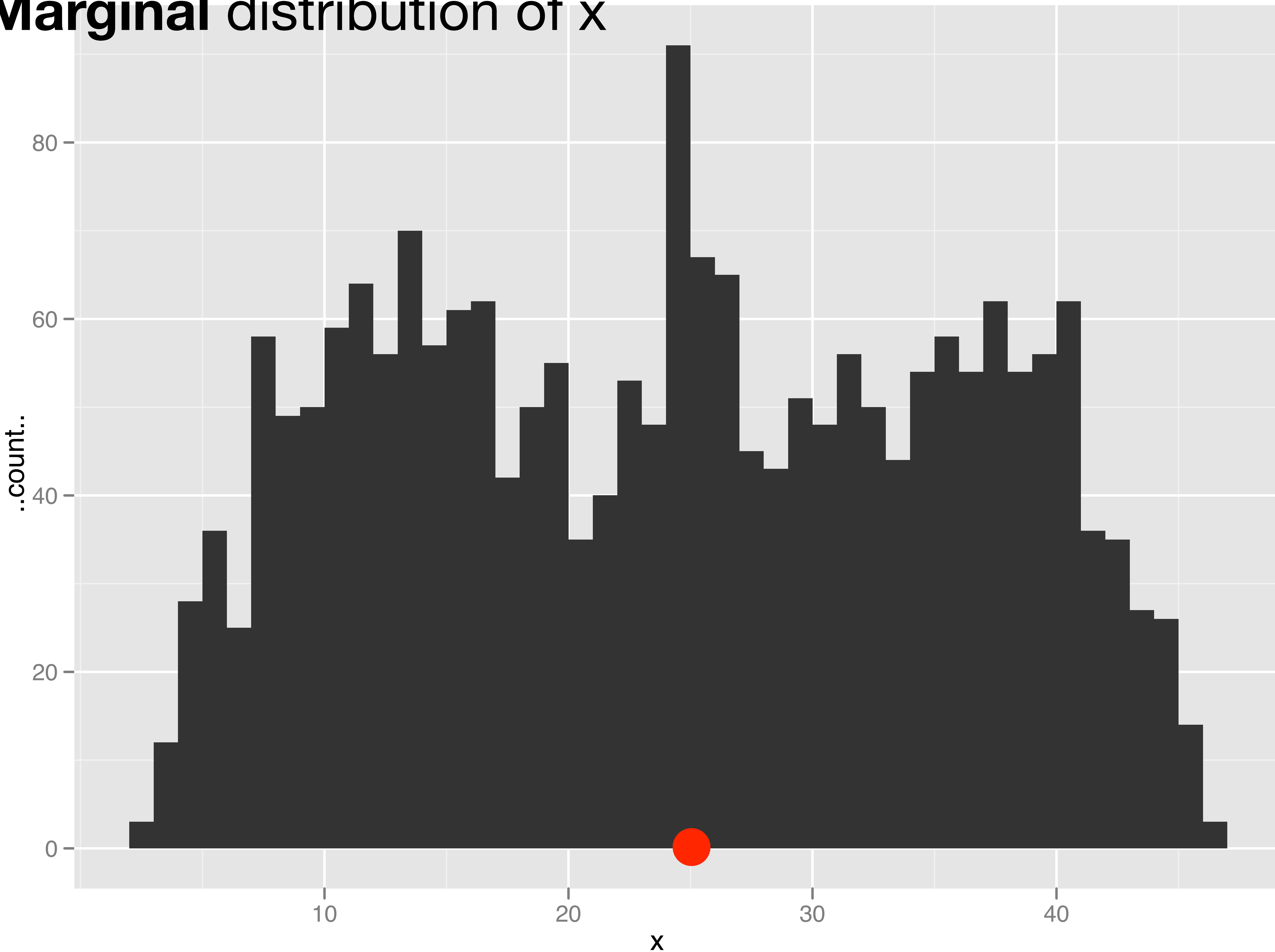
fixing value

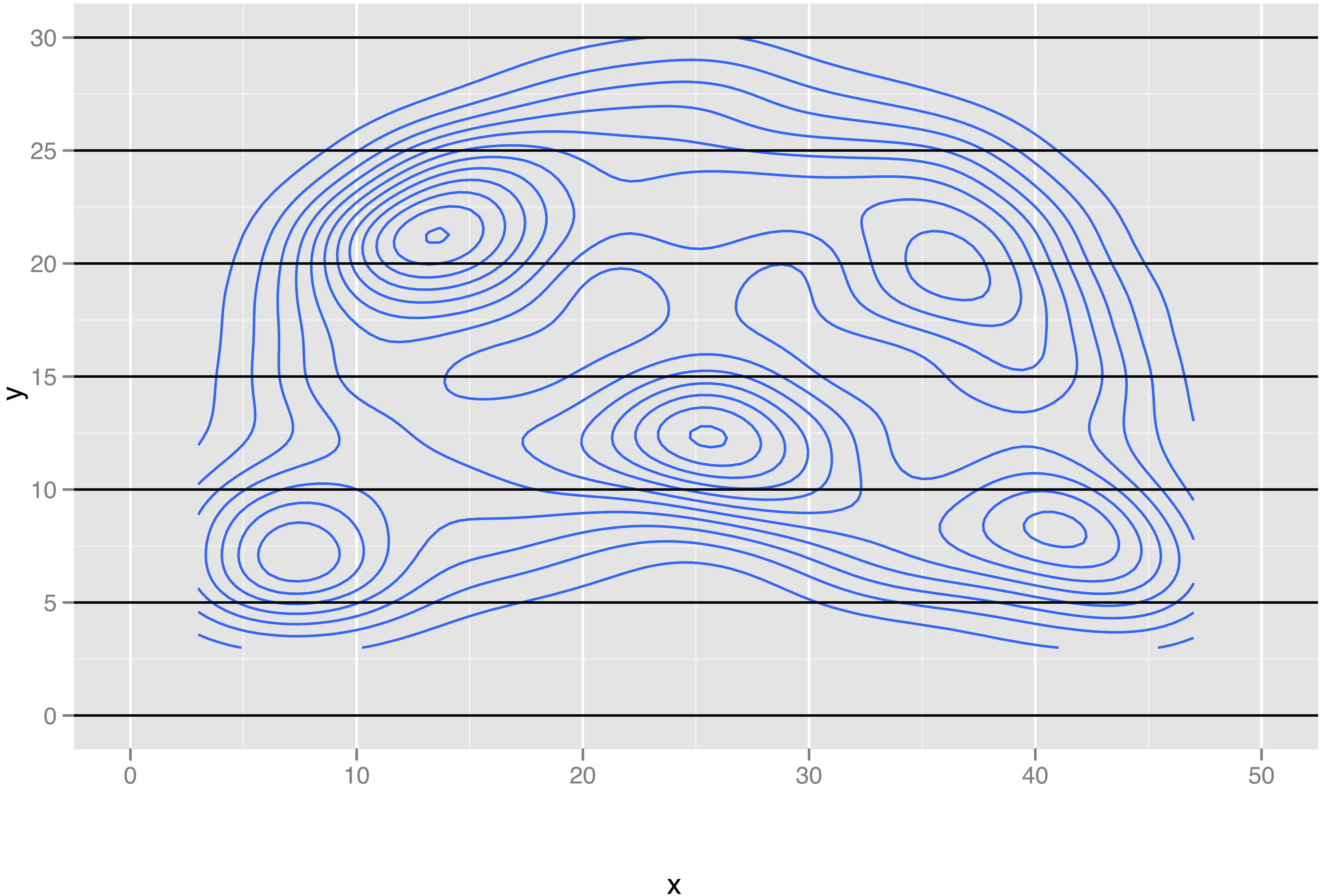
Conditional

Marginal distribution of y



Marginal distribution of x





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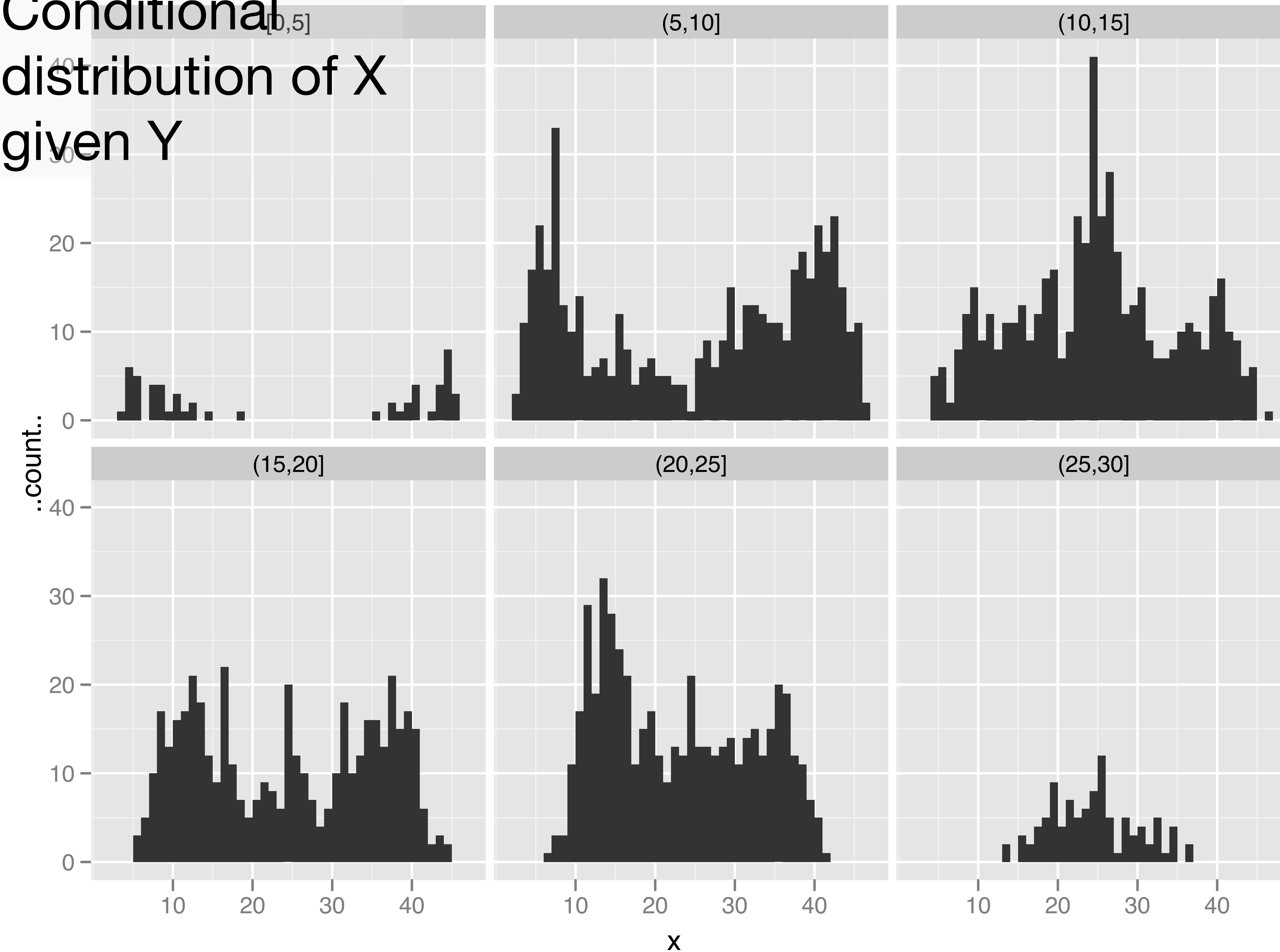
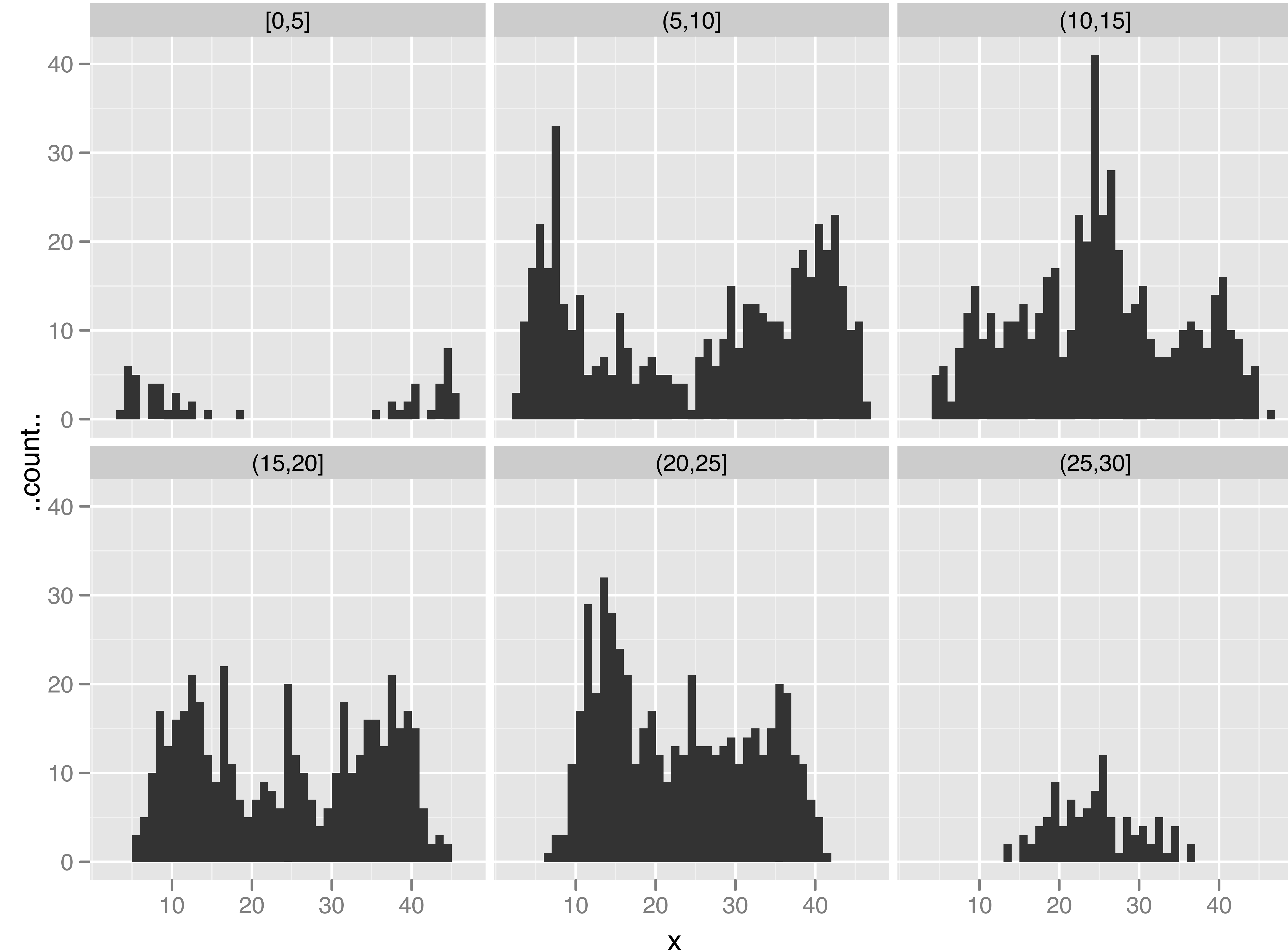
50
x

Conditional distribution of X given Y

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(25,30]
..count..
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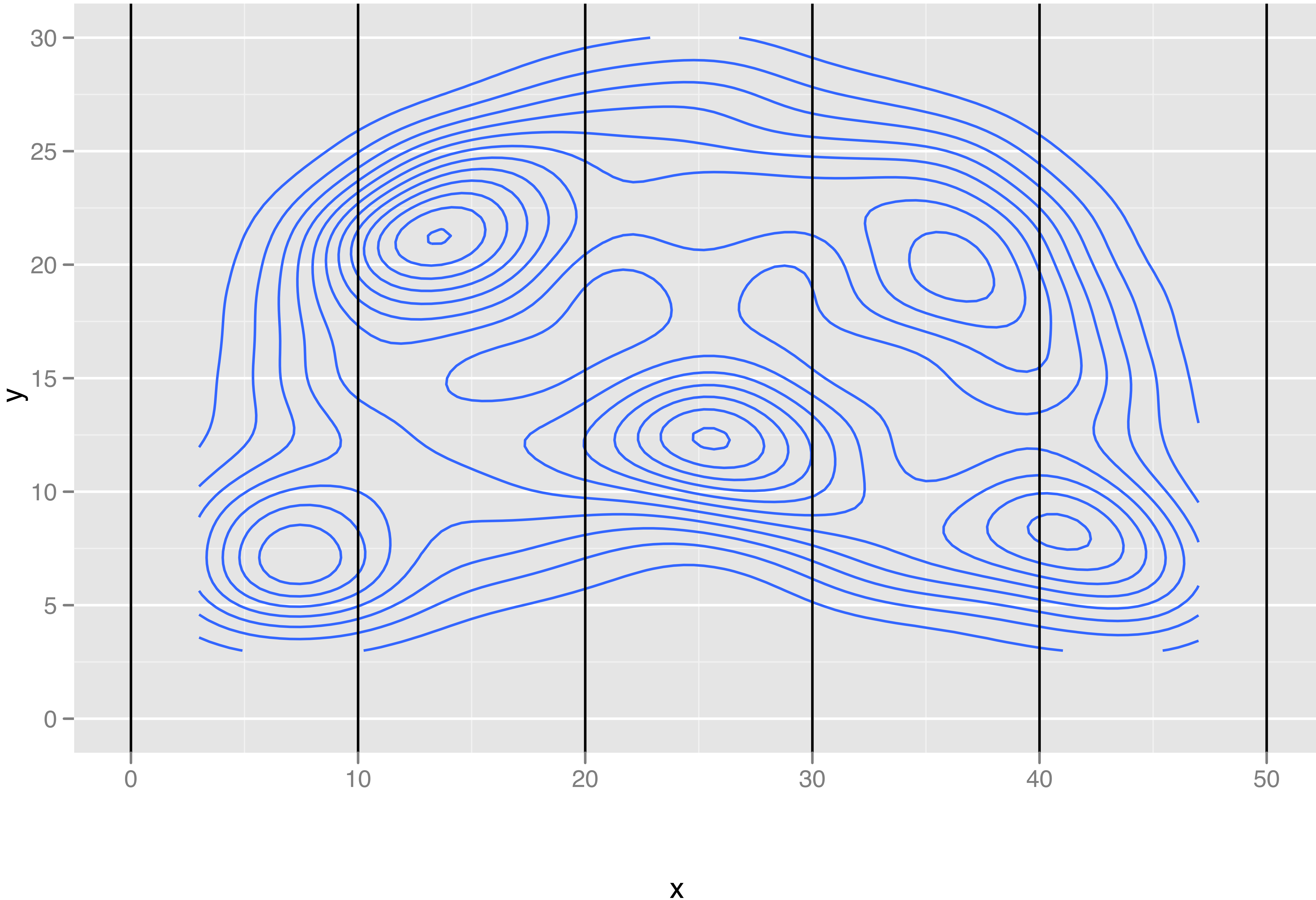
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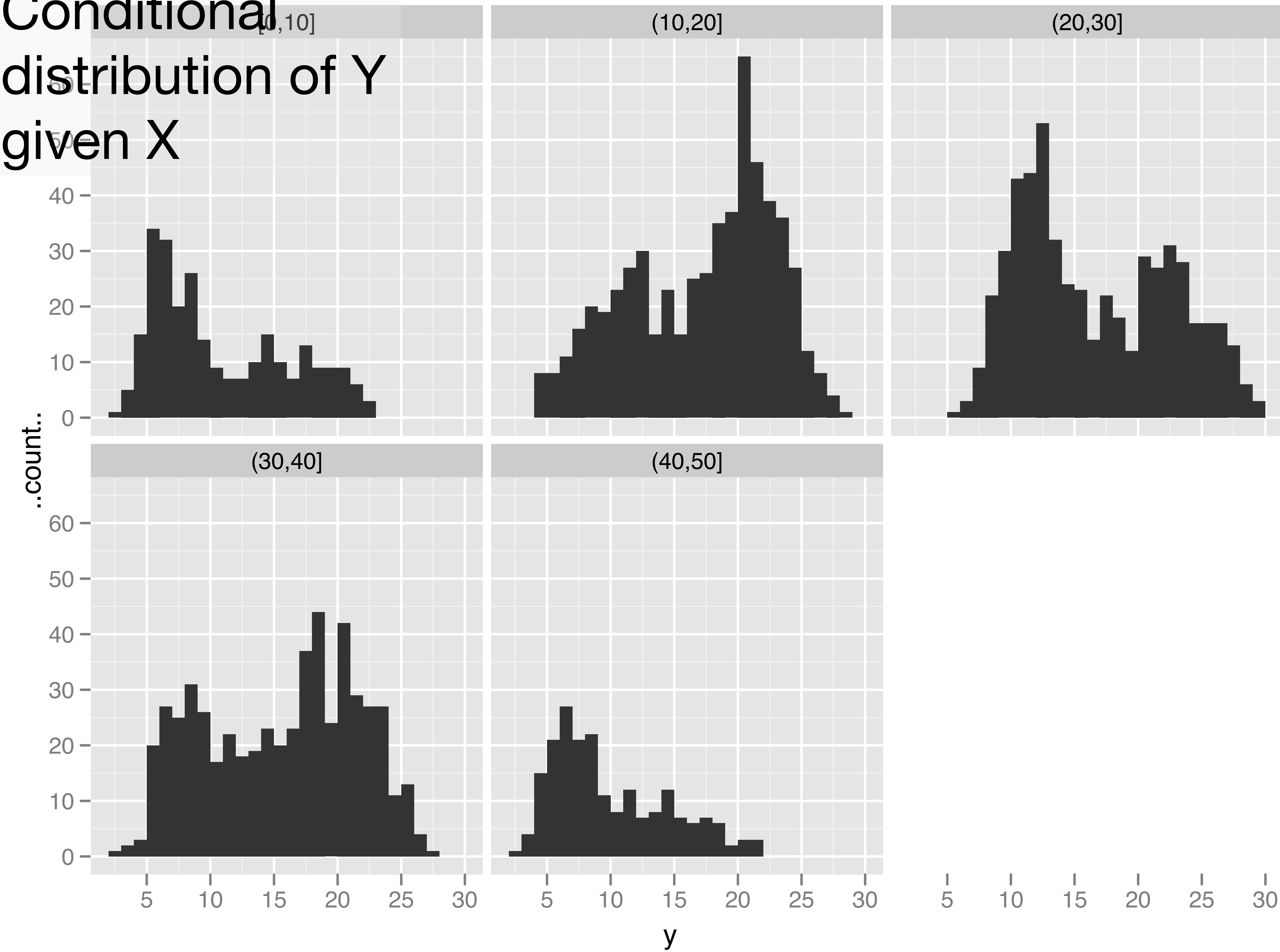
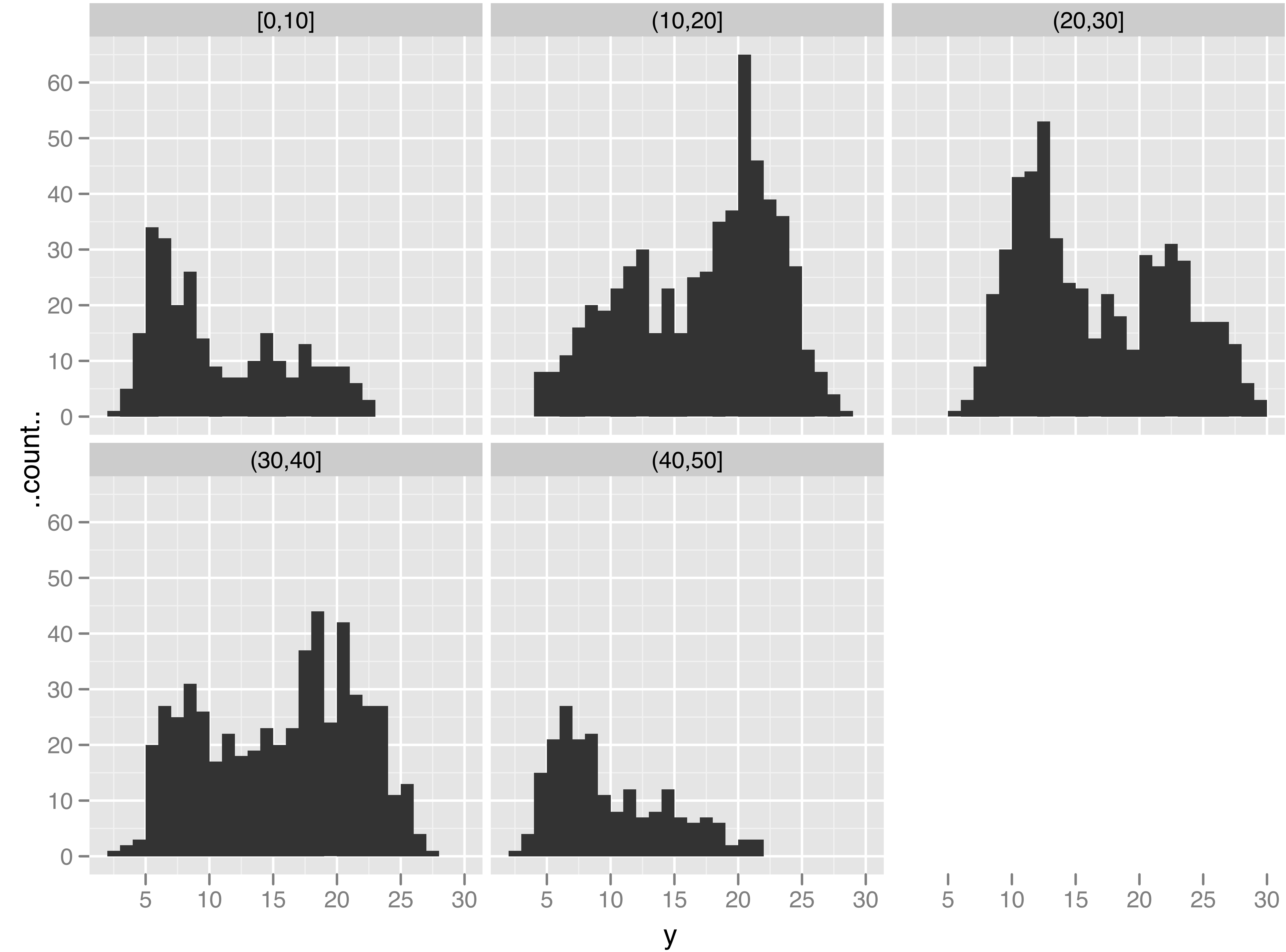
Conditional distribution of Y given X



Thursday, February 16, 12


[0,10]
(10,20]
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(40,50]
..count..
y
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30

Multivariate calculus

Important bits

Partial derivatives (order doesn't matter)

Multiple integrals (order doesn't matter)

2d change of variables - after spring break

Use wolfram alpha:

http://www.wolframalpha.com/examples/Integrals.html

More help

http://www.khanacademy.org/#Calculus

Partial derivatives 1-2

Double integrals 1-6

Next week

I’ll be in Germany, and you’ll be in the capable hands of Garrett.