$$E[(X-E[X])(Y-E[Y])]$$

$$= E[XY - E[X]Y - \cancel{X}E[\cancel{Y}] + E[X]E[Y]]$$

$$= E[XY] - E[X]\cancel{E[Y]} - E[Y]\cancel{E[X]} + E[X]E[Y]$$

$$= E[XY] - E[X]E[Y]$$

If independent:

$$E(XY) = \iint_{\mathbb{R}^2} xy f(x, y)\, dx\, dy$$

$$= \iint_{\mathbb{R}^2} xy\, f(x) f(y)\, dx\, dy$$

$$= \int_{\mathbb{R}} y f(y) \left(\int_{\mathbb{R}} x f(x)\, dx \right) dy$$

$$= E(X) \int_{\mathbb{R}} y f(y)\, dy = E(X)E(Y)$$

so if indep $\operatorname{Cov}(X,Y) = E(X)E(Y) - E(X)E(Y)$
$= 0.$

$E(X)=0 \quad E(X^2)=10 \quad E(X^3)=0$

$Y = X^2$

Y & X can't be independent b/c if you know X then you know what Y is.

$$\operatorname{Cor}(X,Y) = \frac{\operatorname{Cov}(X,Y)}{\sqrt{\operatorname{Cov}(X,X)\operatorname{Cov}(Y,Y)}} = \cancel{\frac{\operatorname{Cov}(X,X)}{\sqrt{\operatorname{Var}(}}}$$

$$= \frac{E(XY)}{\sqrt{E(XX)E(YY)}} = \frac{E(X^3)}{\sqrt{E(X^2)E(X^4)}}$$

$$= \frac{0}{\sqrt{10 \cdot 4}} = 0 \quad \square$$

Generalisation:

* known cdf: $\quad f_A(a) = 2F(x) f(x)$

* $n \geq 2 \quad f_A(a) = n F(x)^{n-1} f(x)$

$A = \dfrac{X-Y}{Y}$ $\quad B = X \hookrightarrow [0,4]$

$\hookrightarrow$ percentage loss $\Rightarrow [0,1]$

$u_1(x,y) = \dfrac{x-y}{y} = a$

$u_2(x,y) = x = b$

$a = \dfrac{b-y}{y} \Rightarrow ay = b-y$

$b = ay + y$

$= y(a+1)$

$\Rightarrow y = \dfrac{b}{a+1}$

$$J = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial x}{\partial b} \\ \frac{\partial y}{\partial a} & \frac{\partial y}{\partial b} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ \frac{-b}{(a+1)^2} & \frac{1}{a+1} \end{vmatrix}$$

$$= \frac{b}{(a+1)^2}$$

$$|J| = \frac{b}{(a+1)^2}$$

$f_{A,B}(a,b) = f_{X,Y}(v_1(a,b),\ v_2(a,b))\ |J|$

$= f_{X,Y}\left(b, \dfrac{b}{a+1}\right) \dfrac{b}{(a+1)^2}$

$= \dfrac{1}{8} \dfrac{b}{(a+1)^2}$

$= \dfrac{b}{c_1} \cdot \dfrac{c_2}{(a+1)^2}$ $\quad$ some $\dfrac{c_2}{c_1} = \dfrac{1}{8}$

$= f(b)\ f(a) \Rightarrow$ independent.