$$x_1 + x_2 + \dots + x_n = \sum_{i=1}^{n} x_i \quad \left(= \sum x_i\right)$$

$$x_1 \times x_2 \times \dots \times x_n = \prod_{i=1}^{n} x_i \quad \left(= \prod x_i\right)$$

use this form if you get stuck/confused

$$\prod x_i y_i = \left(\prod x_i\right)\left(\prod y_i\right)$$

$$\prod (x_i + y_i) \neq \left(\prod x_i\right) + \left(\prod y_i\right)$$

$$\prod c x_i = c^n \left(\prod x_i\right)$$

$$\bar{X}_n = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$\bar{X}_1 = X_1 \quad \bar{X}_2 = \frac{X_1 + X_2}{2} \quad \bar{X}_3 = \frac{X_1 + X_2 + X_3}{n} \quad \dots$$

LLN:

$$\lim_{n\to\infty} \bar{X}_n = E(X) = \mu$$

(basic idea)

BUT r.v number

$$\lim_{n\to\infty} P(|\bar{X}_n - \mu| \geq \varepsilon) = 0 \quad \forall \varepsilon > 0$$

$$P(|\bar{X}_n - \mu| \geq \varepsilon) \to 0$$

$$P(|\bar{X}_n - \mu| < \varepsilon) \to 1$$

or

$$X_n \xrightarrow{P} \mu$$

converges in probability

Many possible limits

conv. in prob	$\lim_{n\to\infty} P(\lvert X_n - X\rvert \geq \varepsilon) = 0$	$\forall \varepsilon > 0$
conv. in dist	$\lim_{n\to\infty} F_n(x) - F(x) = 0$	$\forall x \in S$
almost sure conv.	$P(\lvert\lim_{n\to\infty} X_n - X\rvert \geq \varepsilon) = 0$	$\forall \varepsilon > 0$

WLLN $\lim_{n\to\infty} P(|X_n - \mu| \geq \varepsilon) = 0$

SLLN $P(|\lim_{n\to\infty} X_n - \mu| \geq \varepsilon) = 0$

PROOF OF WLLN:

$$Var(\bar{X}_n) = Var\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n^2} Var(\sum X_i)$$

$$= \frac{1}{n^2}\sum Var(X_i) = \frac{1}{n^2}\cdot n\sigma^2 = \frac{\sigma^2}{n}$$

$$P(|X_n - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{n\varepsilon^2}$$ Chebyshev

$$K\sigma = \varepsilon$$

$$\Rightarrow K = \frac{\varepsilon}{\sigma}$$

$$\lim_{n\to\infty} \frac{\sigma^2}{n\varepsilon^2} = 0$$

$$\lim_{n\to\infty} P(|X_n - \mu| \geq \varepsilon) = 0 \quad \square.$$

EXAMPLE

$Y_i = 1$ if roll a six, 0 otherwise

$Y_i \overset{iid}{\sim}$ Bernoulli(p)

$$\lim_{n\to\infty} \bar{Y}_n \overset{P}{=} E(Y_1) = p$$

So compute an average of all the 1s & 0s and as n gets bigger you'll get closer & closer to p.