

$X_i \stackrel{\text{iid}}{\sim} \text{Binomial}(n, p)$
↑ known ↓ unknown

$$f(x_i) = \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$$

$$f(x_1, x_2, \dots, x_m) = \prod_{i=1}^m \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$$

↑ where I went wrong in class

$$= \left(\prod_{i=1}^m \binom{n}{x_i} \right) p^{\sum_{i=1}^m x_i} (1-p)^{nm - \sum_{i=1}^m x_i}$$

$$\ell(x_1, x_2, \dots, x_m) = \sum_{i=1}^m \ln \binom{n}{x_i} + \ln p \cdot \sum_{i=1}^m x_i + (nm - \sum_{i=1}^m x_i) \cdot \ln(1-p)$$

$$= \sum_{i=1}^m \ln \binom{n}{x_i} + \ln p \cdot m \bar{x}_m + (nm - m \bar{x}_m) \ln(1-p)$$

$$\frac{\partial \ell}{\partial p} = \frac{m \bar{x}_m}{p} - \frac{nm - m \bar{x}_m}{1-p} = 0$$

$$(1-p) m \bar{x}_m = p(nm - m \bar{x}_m)$$

$$\bar{x}_m - p \bar{x}_m = pn - p \bar{x}_m$$

$$\hat{p} = \frac{\bar{x}_m}{n} \quad \square$$

so we basically end up in the same place but we're averaging over the m observations