

Method of moments

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad \text{sample mean}$$

$$s_n^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad \text{sample variance.}$$
$$\left(= \frac{\sum x_i^2}{n} - \bar{x}^2 \right)$$

$$E(x)$$

$$\text{Var}(X) = E((x^* - E(x))^2)$$
$$= E(x^2) - E(x)^2$$

GAMMA

$$E(x) = \alpha \theta \quad \text{Var}(x) = \alpha \theta^2$$

$$\frac{\text{Var}(x)}{E(x)} = \frac{\alpha \theta^2}{\alpha \theta} = \theta \Rightarrow \hat{\theta} = \frac{s_n^2}{\bar{x}}$$

$$\frac{E(x)^2}{\text{Var}(x)} = \frac{\alpha^2 \theta^2}{\alpha \theta^2} = \alpha \Rightarrow \hat{\alpha} = \frac{\bar{x}^2}{s_n^2}$$

YOUR TURN: POISSON

$$E(x) = \lambda \Rightarrow \hat{\lambda} = \bar{x} \quad \text{which is better?}$$
$$\text{Var}(x) = \lambda \Rightarrow \hat{\lambda} = s_n^2$$

UNIFORM

$$E(x) = \sigma/2 \Rightarrow \hat{\theta} = 2\bar{x}$$

$$\bar{x} = \frac{3+5+6+18}{4} = 8$$

$$\Rightarrow \hat{\theta} = 2 \cdot 8 = 16$$

MAXIMUM LIKELIHOOD

$$f(x_i) = \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$$

$$f(x_1, x_2, \dots, x_n) = \prod_{i=1}^n \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$$

$$= \left(\prod_{i=1}^n \binom{n}{x_i} \right) p^{\sum x_i} (1-p)^{n^2 - \sum x_i}$$

$$l(x_1, x_2, \dots, x_n) = \sum_{i=1}^n \ln \binom{n}{x_i} + \sum x_i \cdot \ln p + (n^2 - \sum x_i) \ln(1-p)$$

$$\frac{\partial l}{\partial p} = \frac{\sum x_i}{p} - \frac{n^2 - \sum x_i}{1-p} = 0.$$

$$\Rightarrow (1-p) \lambda \bar{x} = p n \bar{x} - p \lambda \bar{x}$$

$$\bar{x} - p \bar{x} = p n - p \bar{x}$$

$$\Rightarrow \hat{p} = \frac{\bar{x}}{n}$$

Poisson

$$f(x_i) = \frac{e^{-\lambda} \lambda^{x_i}}{x_i!}$$

$$f(x_1, x_2, \dots, x_n) = \frac{\prod_{i=1}^n e^{-\lambda} \lambda^{x_i}}{\prod_{i=1}^n x_i!}$$

$$= e^{-n\lambda} \lambda^{\sum x_i} \prod_{i=1}^n \left(\frac{1}{x_i!} \right)$$

$$l(x_1, x_2, \dots, x_n) = -n\lambda + \sum x_i \cdot \ln(\lambda) - \sum \ln(x_i!)$$

$$\frac{\partial l}{\partial \lambda} = -n + \frac{\sum x_i}{\lambda} = 0$$

$$\hat{\lambda} = \frac{\sum x_i}{n} = \bar{x}$$

[Faint handwritten notes and calculations on the right page, including some mathematical expressions and a signature 'Amir' at the bottom right.]