Method of moments

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \text{sample mean}$$

$$s_n^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \quad \text{sample variance.} \quad \left(= \frac{\sum x_i^2}{n} - \bar{x}^2\right)$$

$E(x)$

$$Var(X) = E((X - E(X))^2)$$
$$= E(X^2) - E(X)^2$$

GAMMA

$E(X) = \alpha\theta \qquad Var(X) = \alpha\theta^2$

$$\frac{Var(X)}{E(X)} = \frac{\alpha\theta^2}{\alpha\theta} = \theta \quad \Rightarrow \quad \hat{\theta} = \frac{s_n^2}{\bar{x}}$$

$$\frac{E(X)^2}{Var(X)} = \frac{\alpha^2\theta^2}{\alpha\theta^2} = \alpha \quad \Rightarrow \quad \hat{\alpha} = \frac{\bar{x}^2}{s_n^2}$$

YOUR TURN : POISSON

$E(X) = \lambda \quad \Rightarrow \quad \hat{\lambda} = \bar{x}$

$Var(X) = \lambda \quad \Rightarrow \quad \hat{\lambda} = s_n^2$

which is better?

UNIFORM

$E(X) = \theta/2 \quad \Rightarrow \quad \hat{\theta} = 2\bar{X}$

$$\bar{x} = \frac{3+5+6+18}{4} = 8$$

$\Rightarrow \quad \hat{\theta} = 2 \cdot 8 = 16$

MAXIMUM LIKELIHOOD

$$f(x_i) = \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$$

$$f(x_1, x_2, \dots, x_n) = \prod_{x=1}^{n} \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$$

$$= \left(\prod_{x=1}^{n} \binom{n}{x_i}\right) p^{\sum x_i} (1-p)^{n^2 - \sum_{i=1}^{n} x_i}$$

$$\ell(x_1, x_2, \dots, x_n) = \sum_{i=1}^{n} \ln\binom{n}{x_i} + \sum x_i \cdot \ln p + (n^2 - \sum x_i) \ln(1-p)$$

$$\frac{\partial \ell}{\partial p} = \frac{\sum x_i}{p} - \frac{n^2 - \sum x_i}{1-p} = 0.$$

$\Rightarrow \quad (1-p)\,n\bar{x} = pn^{2} - p\cancel{n}\bar{x}$

$\bar{x} - \cancel{p\bar{x}} = pn - \cancel{p\bar{x}}$

$\Rightarrow \quad \hat{p} = \dfrac{\bar{x}}{n}.$

POISSON

$$f(x_i) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$= e^{-n\lambda}\lambda^{\Sigma x_i}\prod_{i=1}^{n}\left(\frac{1}{x_i!}\right)$$

$$\ell(x_1, x_2, \ldots, x_n) = -n\lambda + \Sigma x_i \cdot \ln(\lambda) - \Sigma \ln(x_i)!$$

$$\frac{\partial \ell}{\partial \lambda} = -n + \frac{\Sigma x_i}{\lambda} = 0$$

$$\hat{\lambda} = \frac{\Sigma x_i}{n} = \bar{x}$$