$X_i \sim \text{iid}(\mu, \sigma^2) \quad i = 1, \ldots, n \qquad S^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n-1}$

NTS $\quad (n-1)\frac{S^2}{\sigma^2} \sim \chi^2(n-1)$

We know

$\sum_{i=1}^{n} \left(\frac{X-\mu}{\sigma} \right)^2 \sim$ ______ because ______ and ______

But we want to know about

$(n-1)\frac{S^2}{\sigma^2} =$ ______ $= \frac{\sum (X_i - \bar{X})^2}{\sigma^2}$

So.

$$\frac{\sum (X_i - \mu)^2}{\sigma^2} = \frac{\sum (X_i - \bar{X} + \bar{X} - \mu)^2}{\sigma^2}$$

$= \sum$ ______ $+ 2\sum$ ______ $+ \sum$ ______

$\downarrow$ 1 $\qquad \downarrow$ 2 $\qquad \downarrow$ 3

$$= \frac{n-1}{\sigma^2} S^2 + 0 + \left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \right)^2$$

$\Rightarrow (n-1)\frac{S^2}{\sigma^2} =$ ______ $-$ ______

$\quad$ $\overset{\shortparallel}{A} \sim$ ______ $\qquad \overset{\shortparallel}{B} \sim$ ______

$\Rightarrow (n-1)\frac{S^2}{\sigma^2} \sim$ ______ $\quad \square$ ______

① $$\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} = \_\_\_\_\_\_\_\_\_\_ = \frac{s^2}{\sigma^2}(n-1)$$

② $$\sum_{i=1}^{n}(X_i - \bar{X})(\bar{X} - \mu)$$

$= \sum$ ____________________ multiplying out

hint: $n\bar{X} = \sum_{i=1}^{n} X_i$

$=$ ____________________

$=$ ______ $\square$

③ $$\frac{\sum(\bar{X} - \mu)^2}{\sigma^2} = \_\_\_\_\_\_\_\_\_\_$$ sum of constants

moving everything inside the square.

$=$ ______________

$=$ __________ $\square$