

$$X_i \sim \text{iid}(\mu, \sigma^2) \quad i=1, \dots, n$$

$$S^2 = \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{n-1}$$

$$\text{NTS} \quad (n-1) \frac{S^2}{\sigma^2} \sim \chi^2(n-1)$$

We know

$$\sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma} \right)^2 \sim \underline{\hspace{2cm}}$$

because

$$\underline{\hspace{2cm}}$$

and

$$\underline{\hspace{2cm}}$$

But we want to know about

$$(n-1) \frac{S^2}{\sigma^2} = \underline{\hspace{2cm}}$$

=

$$\frac{\sum (X_i - \bar{X})^2}{\sigma^2}$$

So,

$$\sum \frac{(X_i - \mu)^2}{\sigma^2} = \sum \frac{(X_i - \bar{X} + \bar{X} - \mu)^2}{\sigma^2}$$

$$= \sum \underline{\hspace{2cm}} + 2 \sum \underline{\hspace{2cm}} + \sum \underline{\hspace{2cm}}$$

↓ ①
↓ ②
↓ ③

$$= \frac{n-1}{\sigma^2} S^2 + 0 + \left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \right)^2$$

$$\Rightarrow (n-1) \frac{S^2}{\sigma^2} = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$$

" A ~
" B ~

$$\Rightarrow (n-1) \frac{S^2}{\sigma^2} \sim \underline{\hspace{2cm}} \quad \square \quad \underline{\hspace{2cm}}$$

$$\textcircled{1} \sum_{i=1}^n \frac{(x_i - \bar{x})^2}{\sigma^2} = \underline{\hspace{4cm}} = \frac{s^2}{\sigma^2} (n-1)$$

$$\textcircled{2} \sum_{i=1}^n (x_i - \bar{x})(\bar{x} - \mu)$$

$$= \sum \underline{\hspace{4cm}}$$

$$= \underline{\hspace{4cm}}$$

$$= \underline{\hspace{4cm}} \square$$

multiply out

$$\text{hint: } n\bar{x} = \sum_{i=1}^n x_i$$

$$\textcircled{3} \sum \frac{(\bar{x} - \mu)^2}{\sigma^2} = \underline{\hspace{4cm}}$$

$$= \underline{\hspace{4cm}}$$

$$= \underline{\hspace{4cm}} \square$$

sum of constants

moving everything inside the square.