Discrete Distributions | pmf / pdf support mgf / cdf | Continuous Distributions

Geometric (p) $p \in [0,1]$

\# failures until 1 success

$(1-p)^x p$ = pmf Mean = $\frac{1-p}{p}$

$\{0,1,2,\dots\}$

$\frac{p}{1-(1-p)e^t}$ = mgf variance = $\frac{1-p}{p^2}$

Discrete uniform (a,b) $a,b \in S$

Equally likely events

$1/n$, $n = b-a+1$ Mean = $\frac{a+b}{2}$

$\{a, \dots b\}$ variance =

Bernoulli (p) $p \in [0,1)$

Two outcomes, 0 = F, 1 = S

$1-p \quad x=0$ Mean = p

$p \quad x=1$ variance = $p(1-p)$

$\{0,1\}$

$1-p+pe^t$

Negative Binomial (n,p) $r = 1 \dots$ $p \in [0,1]$

\# of failures until r successes

\# of successes, r failures

\# of trials r succ / r fail

$\{0,1,\dots\}$ Mean = $r\left(\frac{p}{1-p}\right)$

$\left(\frac{1-p}{1-pe^t}\right)^r$ variance = $r\left(\frac{p}{(1-p)^2}\right)$

Binomial (n,p) $n = 1,2,\dots$ $p \in [0,1]$

ccess in n Bernoulli trials

$(1-p)^{n-x}$ Mean = np

$\dots n\}$ variance = $np(1-p)$

$\dots + pe^t)^n$

Poisson (λ) $\lambda > 0$ → $(t\lambda)$ ← time

\# of events w/ rate λ
- ind of another
- not at same time
- linearity

$\frac{\lambda^x e^{-\lambda}}{x!}$ Mean = λ

variance = λ

$\{0,1,\dots\}$

$e^{\lambda(e^t-1)}$

Uniform (a,b) $a,b \in \mathbb{R}$

Likelihood event prop. to event

$\frac{1}{b-a}$ = pdf Mean = $\frac{a+b}{2}$

$[a,b]$ variance = $\frac{(b-a)^2}{12}$

$\frac{x-a}{b-a}$ = cdf

Exponential (β) $\beta > 0$ $\beta = 1/\lambda$

Waiting time until a Poisson event

$\lambda e^{-\lambda x}$ Mean = β

$[0,\infty)$ variance = β^2

$1 - e^{-\lambda x}$ mgf = $\frac{1}{1-\beta t}$

(α, θ) $\alpha > 0$, $\theta > 0$

α Poisson events

$\frac{\theta^{-\alpha}}{(\alpha-1)!} x^{\alpha-1} e^{-x/\theta}$

$[0,\infty]$ Mean = $\alpha\theta$

variance = $\alpha\theta^2$

No cdf

mgf = $\frac{1}{(1-\theta t)^\alpha}$

(+) The information in each section is systematically organized.

(+) Purple, green and orange are used consistently to represent pmf/pdf, support and mgf, respectively

(+) Symbols are defined

Continuous : $\int$ pdf **Valid**
- $f(x) \geq 0$
- $\sum_{x \in S} f(x) = 1$
- $\int_{\mathbb{R}} f(x)dx = 1$

Discrete : Σ pmf

integrate $-\infty$ to x: pdf $f(x)$ → $F(x)$ CDF; differentiate: $F(x)$ → $f(x)$

$E(x)$ = expected value = mean

$Var(x) = E(x^2) - E(x)^2$

$E(x^2) = \sum x^2 f(x)$
$\int x^2 f(x)$

Discrete

- Σ pmf
- $E(x) = \sum x f(x)dx$
- $M_x(t) = E(e^{xt}) = \sum e^{xt} f(x)dx$

Continuous

- $\int$ pdf
- $E(x) = \int_{\mathbb{R}} x f(x)dx$
- $M_x(t) = \int e^{xt} f(x)dx = E(e^{xt})$

MGF

moment → $M_x(t) = E(e^{xt})$
- $\frac{d}{dt} M_x(0) = E(x)$ ← 1st moment
- $\frac{d^2}{dt^2} M_x(0) = E(x^2)$ ← 2nd moment

CDF

$F(x) = P(X \leq x) = \int f(t)dt$

☆ PDF NOT A PROBABILITY! ☆ INTEGRATE

Probability

2 conditions for a random experiment
It is ① uncertain
② repeatable

(+) Lots of whitespace makes page easy to scan

mutually exclusive : $A \cap B = \emptyset$
exhaustive : $A \cup B = S$
partition : exclusive & exhausitive

3 Axioms (regular & conditional)

i) $P(A) \geq 0$ for all $A \subset S$	$P(A \mid C) \geq 0$ for all $A \subset S$
ii) $P(S) = 1$	$P(S \mid C) = 1$
iii) $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \emptyset$	$P(A \cup B \mid C) = P(A \mid C) + P(B \mid C)$ if $A \cap B = \emptyset$

$P(A^c) = 1 - P(A)$	$P(A^c \mid C) = 1 - P(A \mid C)$
$P(A) \leq P(B)$ if $A \subset B$	$P(A \mid C) \leq$ ...
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	$P(A \cup$ ... $+ P(B$ ...

(-) Red is not used consistently throughout the document. What does it signify?

independence (A,B)
$P(A \mid B) = P(A)$
$\Rightarrow P(A \cap B) = P(A)P(B)$

mutual independence (A,B,C)
$P(A \cap B) = P(A)P(B)$
$P(B \cap C) = P(B)P(C)$ & $P(A \cap B \cap C) = P(A)P(B)P(C)$
$P(A \cap C) = P(A)P(C)$

Bayes Rule : $P(A \mid B) = \dfrac{P(B \mid A)\,P(A)}{P(B)}$

Counting / calculation of prob.

multiplication rule: multiply # of of possible outcomes of events to get total possible ways

Sampling with...

	order	no order
replacement	n^k	$\binom{n+k-1}{k}$ (k: sample size)
no replacement	nP_m n: total # of obj m: # of obj selected	$\binom{n}{m}$

Random variables and Prob. dist'n

discrete r.v → pmf
...df
...mf
② $f(x_i) \geq 0,\ \forall x_i \in S$

For Pdf
① $\int_R f(x) = 1$ ② $f(x) \geq 0,\ \forall x \in R$

Properties of cdf
1. $0 \leq F(x) \leq 1$
2. $\lim_{x \to -\infty} F(x) = 0$, and $\lim_{x \to \infty} F(x) = 1$
3. non decreasing, right continuous function.

Moments and mgf

...ation of a function
...$) = \sum_{x \in S} f(x)g(x)$
...: $E(X)$
...nce: $Var(X) = E((X - E(X))^2) = E(X^2) - E(X)^2$
... $M_X(t) = E(e^{Xt})$

$\frac{d}{dt} M_X(0) = E(X) = \sum_{x \in S} x f(x)$ (pmf) $= \int_R x f(x)\,dx$ (pdf)

$\frac{d^2}{dt^2} M_X(0) = E(X^2)$

(finding mgf: $E(e^{xt}) = \sum_{x \in S} e^{xt} f(x)$ (pmf) $= \int_R e^{xt} f(x)$ (pdf))

Cumulative distribution Function

$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t$ ...

$P(X \in [a,b]) = \int_a^b f(x)\,dx = F$ ...

$f(x) \xrightarrow{\text{integrate } -\infty \text{ to } x} F(x)$
$F(x) \xrightarrow{\text{Differentiate}} f(x)$

(+) Procedural information is provided in a numbered list followed by a worked example. Reminds you how to solve problem

Transformations

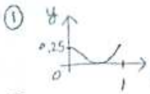
i) Discrete r.v.
Sample space changes, not prob.

ii) Continuous r.v.
If $X \sim normal(\mu, \sigma^2)$, $Y = aX + b$,
$Y \sim normal(a\mu + b, \sigma^2 a^2)$

I. Distribution Function Technique

$Y = u(X)$
① work out range
② Substitute X into cdf of Y
③ Rearrange to get prob. in terms of X
④ Integrate to find cdf
⑤ Differentiate to find pdf

ex) $X = unif(0,1)$, $Y = (X - 0.5)^2$

① 
$y \in [0, 0.25]$

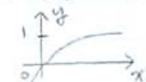
② $F_Y(Y) = P(Y \leq y) = P((x - 0.5)^2 \leq y)$
③ $P(-\sqrt{y} + 0.5 \leq x \leq \sqrt{y} + 0.5)$
④ $\int_{-\sqrt{y}+0.5}^{\sqrt{y}+0.5} 1\,dx = 2\sqrt{y}$
⑤ $f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} 2\sqrt{y} = 1/\sqrt{y}$ $y \in [0, 0.25]$

II. Change of variable technique

Transformation must have an inverse
$Y = u(X) \Rightarrow X = v(Y)$
$f_Y(y) = f_X(v(y))\,|v'(y)|$

① identify range
② identify u
③ figure out inverse v, and its derivative
④ formula

ex) $A \sim Exp(1)$, $B = 1 - e^{-A}$

$f: e^{-x}$
$u: 1 - e^{-x}$

$y \in [0, 1]$

$v: -\ln(1-x)$ $v(y) = -\ln(1-y)$
$v': \frac{1}{1-x}$
$f_Y(y) = e^{\ln(1-y)} \left| \frac{1}{1-y} \right| = 1$ $y \in [0,1]$
$\Rightarrow y \sim uniform(0,1)$

* Relationship to uniform
if $Y \sim uniform(0,1)$, F a cdf
then $X = F^{-1}(Y)$ is a r.v. with cdf $F(x)$

* Mgf of a sum = product of Mgf
* Expectation of a product
= product of expectations (if indep.)

Discrete Distributions	Bernoulli (p)	Discrete Uniform (a,b)	Binomial (n,p)	Negative Binomial (r,p)	Geometric (p)	Poisson (λ)
	an event with 2 outcomes 1 = success, 0 = failure	Equally likely events with integers from a to b	# of success in Bernoulli trials. $n=1,2,3\cdots$; $p\in[0,1]$	# of success in Bernoulli trials until r failures $r=1,2,3,\cdots$ $p\in[0,1]$	# of successes in Bernoulli trials until 1 failure $p\in[0,1]$	count of events with constant rate & independent $\lambda>0$
PMF	$1-p$ $(x=0)$, p $(x=1)$	$\frac{1}{b-a+1}$ → n	$\binom{n}{x}p^x(1-p)^{n-x}$	$\binom{x+r-1}{x}(1-p)^r p^x$	$(1-p)^x p$	$\frac{\lambda^x e^{-\lambda}}{x!}$
MGF	$1-p+pe^t$		$(1-p+pe^t)^n$	$\left(\frac{1-p}{1-pe^t}\right)^r$	$\frac{p}{1-(1-p)e^t}$	$e^{\lambda(e^t-1)}$
mean	p	$\frac{a+b}{2}$	np	$r\frac{p}{1-p}$	$\frac{1-p}{p}$	λ ← rate
variance	$p(1-p)$	$\frac{n^2-1}{12}$	$np(1-p)$	$r\frac{p}{(1-p)^2}$	$\frac{1-p}{p^2}$	λ
Support	$\{0,1\}$	$\{a,a+1,\cdots,b-1,b\}$	$\{0,1,\cdots n\}$	$\{0,1,2\ldots\}$	$\{0,1,2,\cdots\}$	$\{0,1,2\cdots\}$
ex)	Tossing a coin	• getting a grade b/w 5 ~ 15 using a spinner		• # of failures before n successful shots.		① non-overlapping independent intervals ② linearity ③ events don't occur at the same time (+) $X\sim Poisson(\lambda)$, $Y=t(X)$ ⇒ $Y\sim Poisson(\lambda t)$ • Earthquakes, volcanos • switchboard

(+) Examples help you remember what a distribution means

Continuous Distributions	Uniform (a,b)	Exponential (θ)	Exponential (λ)	Gamma (α,θ)	Gamma (α,β)	Normal (μ,σ²)
	Likelyhood of event proportional to its length a,b∈R	waiting time until a Poisson event with average waiting time θ>0	waiting time until a Poisson event with rate λ>0	waiting time for α Poisson events with average waiting time θ>0	waiting time for α Poisson events with rate β>0.	μ∈R, σ²>0
PDF	$\frac{1}{b-a}$	$\frac{1}{\theta}e^{-\frac{x}{\theta}}$	$\lambda e^{-x\lambda}$	$\frac{\theta^{-\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\frac{x}{\theta}}$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$	$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
CDF	$\frac{x-a}{b-a}$	$1-e^{-\frac{x}{\theta}}$		no closed form		$\Phi(x)$
		mgf $\frac{1}{1-\theta t}$		$\frac{1}{(1-\theta t)^\alpha}$		$e^{(\mu t+\frac{1}{2}\sigma^2t^2)}$
mean	$\frac{a+b}{2}$	mean θ	mean: $\frac{1}{\lambda}$	$\alpha\theta$	mean: $\frac{\alpha}{\beta}$	μ
variance	$\frac{(b-a)^2}{12}$	var. θ^2		$\alpha\theta^2$		σ^2
support	$[a,b]$	$[0,\infty)$		$[0,\infty)$		$(-\infty,\infty)$
ex)		• waiting time for a bus that comes every 5 min on average		• waiting time for 10 light-bulbs to fail ($\alpha=10$, $\theta=\frac{1}{0.01}=100$)		

(+) The use of a grid, underlined primary headings at the top of the page, and white space to chunk information make it easier to locate information on this handwritten sheet. Notes focus on equations rather than definitions.

eidensticker

Continuous

$X \sim uniform(a,b)$

pdf: $\frac{1}{b-a}$ cdf: $\frac{x-a}{b-a}$

$\mu = \frac{a+b}{2}$ $\sigma^2 = \frac{(b-a)^2}{12}$

$X \sim exponential(\theta)$

waiting time until a poisson event

average wait time θ

pdf: $\left(\frac{1}{\theta}\right)e^{-\frac{x}{\theta}}$ cdf: $1-e^{-\frac{x}{\theta}}$

MGF: $\frac{1}{1-\theta t}$ $\mu=\theta$ $\sigma^2=\theta^2$

$X \sim Gamma(\alpha,\theta)$

Waiting time til α events

avg. wait time θ

pdf: $\left(\frac{\theta^{-\alpha}}{\Gamma(\alpha)}\right)x^{\alpha-1}e^{-\frac{x}{\theta}}$

MGF: $\frac{1}{(1-\theta t)^{\alpha}}$

$\mu=\alpha\theta$ $\sigma^2=\alpha\theta^2$

$X \sim Normal(\mu,\sigma^2)$

pdf: $\left(\frac{1}{\sqrt{2\pi\sigma}}\right)e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$

cdf: $\Phi(x)$ MGF: $E\left(\mu t+\frac{1}{2}\sigma^2 t^2\right)$

if $X \sim Normal(\mu,\sigma^2)$, $Y=aX+b$

$\Rightarrow Y \sim Normal(a\mu+b, a^2\sigma^2)$

[If $Y=u(X)$ & $v(y)$ is inverse of u,
$X=v(Y)$
Then $f_Y(y)=f_X(v(y))|v'(y)|$]

$z=\frac{x-\mu}{\sigma}$ $P(Z<z)=\Phi(z)$

$\Phi(0)=.5$, $\Phi(-x)=1-\Phi(x)$

If $Y \sim Uniform(0,1)$ & $F(y)$ is a cdf

Then $X=F^{-1}(Y)$ is a r.v. w/ cdf $F(x)$

Discrete

$X \sim uniform(a,b)$

pmf $=\frac{1}{n}$, $n=a-b+1$

$\mu=\frac{a+b}{2}$ $\sigma^2=\frac{n^2-1}{12}$

Bernoulli(p)

An event w/ two outcomes

pmf: $1-p$ $x=0$ failure; p $x=1$ success

MGF: $1-p+pe^t$

$\mu=p$, $\sigma^2=p(1-p)$

Binomial(n,p)

# of successes in n Bernoulli trials

pmf: $\binom{n}{x}p^x(1-p)^{n-x}$

MGF: $(1-p+pe^t)^n$

$\mu=np$ $\sigma^2=np(1-p)$

$X \sim Geometric(p)$

# of failures until 1 success

pmf: $(1-p)^x p$

MGF: $\frac{p}{1-(1-p)e^t}$

$\mu=\frac{1-p}{p}$ $\sigma^2=\frac{1-p}{p^2}$

$X \sim NegBin(r,p)$

pmf: $\binom{x+r-1}{x}(1-p)^x p^r$

MGF: $\left(\frac{1-p}{1-pe^t}\right)^r$

# of failures (x) until r successes

$\mu=r\left(\frac{p}{1-p}\right)$

$\sigma^2=r\left(\frac{p}{(1-p)^2}\right)$

$X \sim Poisson(\lambda)$

Count of events that have constant rate(λ), and are independent of one another

pmf: $\frac{\lambda^x e^{-\lambda}}{x!}$ MGF: $E(\lambda(e^t-1))$

$\mu=\lambda$, $\sigma^2=\lambda$

prob = pmf
$f(X=x)=f(x)$

pmf conditions

1) $\sum_{x_i \in S} f(x_i)=1$

2) $f(x_i)\geq 0, \forall x_i \in S$

pdf conditions

1) $f(x)\geq 0, x\in\mathbb{R}$

2) $\int_{-\infty}^{\infty} f(x)dx=1$

differentiate

pdf $f(x)$ $F(x)$ cdf

integral

(+) Minimal words are fine if you think in equations

(-) The information in the columns could be arranged better. For example, x ~ Geometric could be paired alongside x~Exponential and Gamma next to NegBin.

(+/-) Handwriting is generally neat, although some of the terms are a bit small.

(+) Using a table or grid format is a useful way to organize information and allows you to find what you need quickly. But beware spending too much time making a beautiful note sheet instead of quickly capturing the most relevant information

Type:	Name:	Description:	PMF/PDF:	CDF:	Support:	MGF:	Mean:	Variance:
DISCRETE	Discrete Uniform (a,b)	Equally likely events	$\frac{1}{\ }$		{a, a+1, ..., b-1, b}		$\frac{a+b}{2}$	$\frac{n^2-1}{12}$
	Bernoulli (p)				{0,1}	$1-p+pe^t$	p	p(1-p)
	Binomial (n,p)	count)			{0,1, ..., n}	$(1-p+pe^t)^n$	np	np(1-p)
	Geometric (p)	The number of successes in Bernoulli(p) trials until one failure.	$(1-p)^x p$		{0, 1, 2, ...}	$\frac{p}{1-(1-p)e^t}$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
	Negative Binomial (r,p)	The number of successes in Bernoulli(p) trials until r failures.	$\binom{x+r-1}{x}(1-p)^r p^x$		{0, 1, 2, ...}	$(\frac{1-p}{1-pe^t})^r$	$r\frac{p}{1-p}$	$r\frac{p}{(1-p)^2}$
	Poisson (λ)	Count of events that have constant rate, and are independent of one another.	$\frac{\lambda^x e^{-\lambda}}{x!}$		{0, 1, 2, ...}	$\exp(\lambda(e^t-1))$	λ	λ
CONTINUOUS	Uniform (a, b)	Likelihood of event proportional to its length.	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	[a,b]		$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
	Exponential (θ)	Waiting time until a Poisson event with average waiting time θ	$\frac{1}{\theta}e^{\frac{-x}{\theta}}$	$1-e^{\frac{-x}{\theta}}$	[0, ∞)	$\frac{1}{1-\theta t}$	θ	θ^2
	Exponential (λ)	Waiting time until a Poisson event with rate λ	$\lambda e^{-x\lambda}$				$\frac{1}{\lambda}$	
	Gamma (α, θ)	Waiting time for α Poisson events with average waiting time θ	$\frac{\theta^{-\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{\frac{-x}{\theta}}$	No closed form	[0, ∞)	$\frac{1}{(1-\theta t)^\alpha}$	αθ	$\alpha\theta^2$
	Gamma (α, β)	Waiting time for α Poisson events with	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$				$\frac{\alpha}{\beta}$	
	Norm (μ,σ		$\mu)^2$	Φ(x)	(-∞, ∞)	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$	μ	σ^2

(+) Good structure to show similarities between regular and conditional probabilities

Rules:				
	Regular	$P(A) \geq 0$	$P(S) = 1$	$P(A \cup B) = P(A) + P(B)$
	Conditional	$P(A\|C) \geq 0$	$P(S\|C) = 1$	$P(A \cup B \| C) = P(A\|C) + P(B\|C)$

Formulae:				
	Regular	$P(A') = 1 - P(A)$	$P(A) \leq P(B)$ if $A \subset B$	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Conditional	$P(A'\|C) = 1 - P(A\|C)$	$P(A\|C) \leq P(B\|C)$ if $A \subset B$	$P(A \cup B \| C) = P(A\|C) + P(B\|C) - P(A \cap B \| C)$

STAT 310—Test 1 Comprehensive Review Sheet

(+) This review sheet includes a lot of definitions stated in words. Good idea if you're a verbal thinker!

Definitions

Probability - tool for dealing with uncertain events

Axiomatic definition (for any probability function)

1. $P(A) \geq 0$, for all $A \subset S$
2. $P(S) = 1$
3. $P(A \cup B) = P(A) + P(B)$ if $A \cap B = 0$

Random experiment - trial/observation that is uncertain and repeatable

Sample space - set of all possible **outcomes**. Called S or U.

Countably infinite - can be modeled by the natural numbers (vs. **Uncountably infinite**)

Event - subset of sample space

Mutually exclusive <=> A n B = empty set

Exhaustive - events span the entire sample space

Partition - mutually exclusive and exhaustive

Probability Basics

(+) Use of headings, bold, underlining, indentation, alignment, and white space signal the hierarchical relationships between chunks of information. Font is easy to read.

$P(A') = 1 - P(A)$

If $A \subset B$, then $P(A) \leq P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B$

Conditional probability

$P(A \mid B) = P(A \cap B) / P(B)$

or $P(A \cap B) = P(A \mid B) P$

Law of Total Probability

For partition B, the total probability of A equals the sum of all P(A|Bi)P(Bi) for each condition Bi in B

$$\Pr(A) = \sum_n \Pr(A \mid B_n) \Pr(B_n)$$

Toolbox

Complements, Unions to sums (if mutually exclusive or subtract intersection), Intersection <-> conditioning, Intersection <-> product (if independent), Law of total probability, Bayes' Rule

Calcul

Equall

Count

Multip total possibilities ; Sampling with replacement - n^r ; Sampling without replacement (1) Permutation (order matters)

$$\frac{n!}{(n-k)!}$$

(2) Combination (order doesn't matter; equals permutation / number of orderings)

$$\frac{n!}{k!(n-k)!}$$

Steps:

(1) Find size of sample space |S| (2) Find size of event |E| (3) Find |E|/|S

ent - $P(A \cap B) = P(A)P(B)$, so $P(A \mid B) = P(A)$

ndep. - Each pair in the set is independent; P

P(A)P(B), P(A ∩ C) = ..

ndependent - Every combination of events in independent. Pairwise + P(A ∩ B ∩ C) = P(A)P

Bayes' Rule

(+) Good idea to also capture strategies, not just facts.

P(B | A) P(A) ... d A

P(B) usually r ... ility to find

Set Theory:

A u (B n C) = (A u B) n (A u C)

A n (B u C) = (A n B) u (A n C)

(A' n B')' = A u B (A' u B')' = A n B

Random Variables- random experiment with numeric sample space

support - sample space / range of function

Discrete - finite or countably infinity support

probability mass function (pmf) - f(x)

$P(X = x) = f(x)$

$$P(a < X < b) = \sum_{x_i \in (a,b)} f(x_i)$$

Requires Probabilities sum to 1; All probabilities are positive (or 0)

Continuous - Uncountably infinity support

Probability density function (pdf)

Disitributions - Common pmf's/pdf's

Mean - middle/ balance point; E(X); additive => E(X +Y) = E(X) + E(Y); homogeneous => E(cX) = cE(X); E(c) = c

$$E(g(X)) = \sum_{x \in S} f(x) g(x)$$

Variance - spread = $E[(X - E[X])^2] = E(X^2) - E(X)^2$

i^{th} moment - $E(X^i) = \mu_i'$; mean - first moment

i^{th} central - $E[(X - E(X))^i] = \mu_i$; variance - second central moment

skewness - skewed right if trails off to right

kurtosis - leptikurtic = peak; platikurtic = plateau

moment generating function - $M_x(t) = E(e^{Xt})$

Mx = My <=> fx = fy

mean - Mx'(t) at t = 0

variance - Mx''(t) - Mx'(t)^2 at t = 0

Discrete

Discrete uniform(a, b) a, b ∈ Z

Equally likely events labelled with integers from a to b.

PMF	$1/n \quad n = \cancel{a-b+1}$ b-a+1
Support	$\{a, a+1, \ldots, b-1, b\}$
Mean	$\frac{a+b}{2}$
Variance	$\frac{n^2-1}{12}$

Bernoulli(p) p ∈ [0, 1]

An event with two outcomes:
1 = success, 0 = failure

PMF	$1-p \quad x=0$; $p \quad x=1$
Support	$\{0, 1\}$
MGF	$1-p+pe^t$
Mean	p
Variance	$p(1-p)$

Binomial(n, p) n = 1, 2, 3, ...; p ∈ [0, 1]

The number of successes in n Bernoulli(p) trials (a count)

PMF	$\binom{n}{x} p^x (1-p)^{n-x}$
Support	$\{0, 1, \ldots, n\}$
MGF	$(1-p+pe^t)^n$
Mean	np
Variance	$np(1-p)$

Geometric(p) p ∈ [0, 1] successes

The number of ~~failures~~ in Bernoulli(p) trials until one ~~success.~~ failure.

PMF	$(1-p)^x p$
Support	$\{0, 1, 2, \ldots\}$
MGF	$\frac{p}{1-(1-p)e^t}$
Mean	$\frac{1-p}{p}$
Variance	$\frac{1-p}{p^2}$

Other notes : Sum of binomials with same p -> just add the trials.
Conditions on Poisson : (1) Non-overlapping intervals are independent (2) Linearity/ smoothness (probability of event is proportional to time) (3) Events don't occur at the same time.
Poisson multiplication : Y = tX then Y ~ Poisson (lambda * t).

Negative binomial(r, p) r = 1, 2, 3, ... p ∈ [0, 1]

The number of ~~failures~~ successes in Bernoulli(p) trials until r ~~successes.~~ failures

PMF	$\binom{x+r-1}{x}(1-p)^r p^x$
Support	$\{0, 1, 2, \ldots\}$
MGF	$\left(\frac{1-p}{1-pe^t}\right)^r$
Mean	$r\frac{p}{1-p}$
Variance	$r\frac{p}{(1-p)^2}$

Poisson(λ) λ > 0

Count of events that have constant rate, and are independent of one another.

PMF	$\frac{\lambda^x e^{-\lambda}}{x!}$
Support	$\{0, 1, 2, \ldots\}$
MGF	$\exp(\lambda(e^t-1))$
Mean	λ
Variance	λ

Continuous

Uniform(a, b) a, b ∈ R

Likelihood of event proportional to it's length.

PDF	$\frac{1}{b-a}$
CDF	$\frac{x-a}{b-a}$
Support	$[a, b]$
Mean	$\frac{a+b}{2}$
Variance	$(b-a)^2$ / 1[illegible]

(+) More procedural information about how to solve a problem.

Exponential(θ) [illegible]

Waiting time until a [illegible] average waiting time θ.

PDF	$\frac{1}{\theta}e^{-x/\theta}$
CDF	$1-e^{-x/\theta}$
Support	$[0, \infty)$
MGF	$\frac{1}{1-\theta t}$
Mean	θ
Variance	θ^2

	Until 1	Multiple
Count	Geometric	Negative Binomial
Waiting Time	Exponential	Gamma

Gamma(α, θ) α > 0, θ > 0

Waiting time for a Poisson events with average waiting time θ.

PDF	$\frac{\theta^{-\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-x/\theta}$
CDF	No closed form
Support	$[0, \infty)$
MGF	$\frac{1}{(1-\theta t)^\alpha}$
Mean	$\alpha\theta$
Variance	$\alpha\theta^2$

Normal(μ, σ²) μ ∈ R, σ² > 0

PDF	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
CDF	$\Phi(x)$
Support	$(-\infty, \infty)$
MGF	$\exp(\mu t + \frac{1}{2}\sigma^2 t^2)$
Mean	μ
Variance	σ^2

Continuous—f(x) is not a probability, but a density, integrate to get probability. f(x) can be greater than 1. Conditions : f(x) >= 0 and integral over R is 1

cdf - F(x): P(X <= x) ; integrate or sum; monotone increasing , right continuous

Gamma function Γ(n)= (n-1)!

Normal Distribution μ = the mean; σ^2 = variance; if Y = aX + b then Y ~ Normal(aμ + b, $a^2\sigma^2$) ; standard normal is Z~Normal(0,1); can any to normal via Z = (X - [illegible]

[illegible] Sample space changes, not prob.

Distribution Function Technique
(1) Find range (2) Write out cdf of Y, then substitute with x [ie. P(Y <= y) = P(u(x) <= y)] (3) Solve for probability [ie. Rearrange so that P(x <= v(y))] (4) Integrate using pdf of x to find cdf of Y (5) Differentiate with respect to Y to find pdf

Change of Variable Technique Only if u has inverse v
Y = u(X), X = v(Y) : $f_Y(y) = f_X(v(y))\,|v'(y)|$
Steps: (1) Find range (2) Identify u (3) Figure out v and its derivative (4) Plug into formula

If Y ~ Uniform(0, 1) and F is a cdf, then, X = $F^{-1}(Y)$ is a rv with cdf F(x). If X has cdf F and Y = F(X), then Y ~ Uniform(0, 1)

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