STAT310
Practice Problems - Solutions
Week 1

January 24, 2012

1 Defining sample spaces.

1. (a) $\Omega = \{H, T\}$
   (b) $\Omega = \{3\}$
2. $\Omega = \{HH, HT, TT\}$
3. $\Omega = \{HH, HT, TH, TT\}$
4. Possible outcomes are (1) finite sequences of H that end with a single T, and (2) an infinite sequence of H. In other words, $\Omega = \{T, HT, HHT, HHHT, ..., \{HHH...\}\}$.

2 Unions and intersections I.

Note that the definition of a **set** is such that the ordering of a set does not matter, e.g. $\{1, 2\} = \{2, 1\}$.

1.
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$
   since the event $A \cap B$ is equal to $\{HH\}$.

2.
$$P(A \cap B) = P(HH) = \frac{1}{4}$$

3.
$$\begin{aligned} P(\overline{\bar{A} \cup \bar{B}}) &= 1 - [P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})] \\ &= 1 - [\frac{1}{2} + \frac{1}{2} - \frac{1}{4}] \\ &= 1 - [1 - \frac{1}{4}] \\ &= \frac{1}{4} \end{aligned}$$

since

$$\bar{A} = \{TH, TT\}$$
$$\bar{B} = \{HT, TT\}$$
$$\bar{A} \cap \bar{B} = \{TT\}.$$

3 Unions and intersections II.

1.

$$\begin{aligned} P(B) &= \frac{250 + 150 + 150}{150 + 250 + 50 + 250 + 150 + 150} \\ &= \frac{550}{1000} \\ &= 0.55 \end{aligned}$$

2.

$$\begin{aligned} P(A^C \cap B) &= \frac{150 + 150}{1000} \\ &= 0.30 \end{aligned}$$

since

$$A^C = \{S, R\}$$

3.

$$\begin{aligned} P(A \cup B^C) &= P(A) + P(B^C) - P(A \cap B^C) \\ &= \frac{150 + 250}{1000} + \frac{150 + 250 + 50}{1000} - \frac{150}{1000} \\ &= 0.7 \end{aligned}$$

since

$$B^C = \{yes\ \ tax\ \ increase\}$$

4 Counting.

This is a problem of counting outcomes under the *ordered, without replacement* case. We know the number of possible words formed when the ordering of redundant letters is accounted for, so what we must do is divide out the redundant orderings. The redundant orderings are *E* (3 times), *O* (2 times), and *K* (2 times). Hence the total number of possible outcomes is

$$\frac{10!}{2!2!3!}$$

and the probability of obtaining the word *BOOKKEEPER* is equal to

$$\frac{1}{\frac{10!}{2!2!3!}}.$$

5 The Boy or Girl problem.

This is a tricky problem. One's first inclination is to say that each birth is independent of the other, and that the answer is $\frac{1}{2}$. However, this answer is incorrect.

The sample space, if we were not given information that one child was a boy, would be $\Omega = \{BB, BG, GB, GG\}$. Given that one child is a boy, however, the sample space becomes $\Omega = \{BB, BG, GB\}$. Thus the probability that the other child is a girl, given that one child is a boy, is equal to $\frac{2}{3}$.

6 The Tuesday Birthday problem.

This is a tricky problem as well. In this case, one's first inclination here would be to say that the information about the Tuesday birth is irrelevant, and that the answer is $\frac{1}{2}$. However, this is another case where the most intuitive answer is not correct.

To see why, suppose first that it is the older child who was a son born on a Tuesday. Then the second child could be either a girl or a boy, and could have been born on any of seven days of the week, resulting in a total of 14 possibilities.
Now suppose that it is the younger child who was a son born on a Tuesday. Then the first child could, again, be either a girl or a boy, and could have been born on any of seven days of the week, again resulting in 14 possibilities. Added to our original 14 possibilities, this would seem to give 28 possibilities.
This is where the mistake occurs: one possibility got counted twice. Specifically, we counted twice the outcome that both children were boys born on Tuesdays. Therefore, there are actually only 27 possibilities. Since 13 of them involve the second child being a boy, the probability is equal to $\frac{13}{27}$.