STAT310
Practice Problems
Week 2

February 1, 2012

1 Mutually independent events.

Suppose A, B and C are mutually independent. Prove that the following events are independent:
(a) A and $B \cap C$
(b) A and $B \cup C$
(c) $\bar{A}$ and $B \cap C$
(d) A and $\bar{B} \cap C$

2 Mammography.

About 12% of women will develop some form of invasive breast cancer in their life. In this question, you'll explore the statistics of mammography, a common detection method. The following data comes from Screening for Breast Cancer: Systematic Evidence Review Update for the U. S. Preventive Services Task Force: The probability that woman develops breast cancer increases as she gets older: for women age 40-49 it's 1 in 69; 50-59, 1 in 38, and 60-69, 1 in 27.

Breast cancer is more often incorrectly diagnosed in younger women. The false positive rate per 1000 screenings for women in their 40's is 97.8, in their 50's, 86.6, and in their 60s, 79.0. Breast cancer is also harder to detect in younger women. The false negative rate per 1000 screenings is 1.0 for women their 40's, 1.1 for women in their 50's, and 1.4 for women in their 60's.

(a) Calculate the probability that a randomly selected 40-year-old with a positive mammogram has breast cancer, using natural frequencies.
(b) Calculate the probability that a randomly selected 50-year-old with a positive mammogram has breast cancer. Do the same for a 60-year-old woman. Use Bayes Rule.
(c) At what age would you recommend starting mammogram screenings? Why?
(d) What properties would the test need so that a positive result had a $< 10\%$ chance of being wrong?
(e) What properties would the test need so that a negative result had a $< 10\%$ chance of being wrong?

3 Set Operations.

3.1 Part A

If $P(B) = 0.1$, $P(A^C|B) = 0.85$, and $P(A^C \cap B^C) = 0.3$, find
(a) $P(A \cap B^C)$

(b) $P(A)$
(c) $P(B|A)$

3.2 Part B (general case)

If $P(B) = p$, $P(A^C|B) = q$, and $P(A^C \cap B^C) = r$, find
(a) $P(A \cap B^C)$
(b) $P(A)$
(c) $P(B|A)$

4 Counting.

An urn contains two white balls and two black balls. A number is randomly chosen from the set $\{1, 2, 3, 4\}$, and that number of balls are removed from the urn. Find the probability that the number i, $i = 1, 2, 3, 4$, was chosen if at least one white ball was removed from the urn.