

STAT310  
Practice Problems  
Week 4

February 9, 2012

### 1 Moment generating functions (I).

Find the moment generating function corresponding to

1.  $f(x) = \frac{1}{c}$ ,  $0 < x < c$
2.  $f(x) = \frac{2x}{c^2}$ ,  $0 < x < c$
3.  $f(x) = \frac{1}{2\beta} e^{-|x-\alpha|/\beta}$ ,  $-\infty < x < \infty$ ,  $-\infty < \alpha < \infty$ ,  $\beta > 0$

### 2 Moment generating functions (II).

Identify the distribution that corresponds with the following MGFs.

1.  $M_X(t) = \frac{1}{N} \sum_{i=1}^N e^{it}$
2.  $M_X(t) = (1-p) + pe^t$
3.  $M_X(t) = [pe^t + (1-p)]^n$
4.  $M_X(t) = e^{\lambda(e^t-1)}$
5.  $M_X(t) = \left( \frac{p}{1-(1-p)e^t} \right)^r$
6.  $M_X(t) = \frac{pe^t}{1-(1-p)e^t}$

### 3 Moment generating functions (III).

For each of the following functions, find the MGF,  $E[X]$ , and  $\text{Var}[X]$ .

1.  $f(x) = \frac{1}{2} e^{-|x|}$ ,  $x \in (-\infty, \infty)$
2.  $f(x) = ce^{-x}$ ,  $x \in [0, \infty)$

## 4 Continuous random variables.

For each of the following functions, verify whether the function is a PDF. If it is: (A) Calculate  $P(X > 0)$  and  $P(1 < X \leq 2)$ , using the PDF. (B) Find the CDF. (C) Calculate  $P(X > 5)$  and  $P(1 < X < 2)$ , using the CDF. Is your answer the same as in Part A?

1.  $f(x) = x^3, x \in (-10, 10)$

2.  $f(x) = x^3, x \in (0, 10)$

3.  $f(x) = \frac{x^2}{4}, x \in (-6^{1/3}, 6^{1/3})$

4.  $f(x) = \frac{3}{16}\sqrt{x}, x \in [0, 4]$