

STAT310
Practice Problems
Week 5

March 26, 2012

1 Continuous random variables.

For each of the following random variables, identify the distribution (including the parameters) that most closely matches the situation.

1. Suppose that I am throwing pebbles into a lake, and if I throw my hardest the pebble lands 20 feet away from me upon the first skip. Let X_1 be the distance in centimeters that the pebble lands from me upon the first skip. (1) What is the distribution of X_1 ? (2) What is the mean distance the pebble will land from me upon the first skip? (3) What is the probability that it will land between 10-20 feet away from me?
2. Let X_2 be the mean distance the pebble lands from me upon the first skip, out of every 100 tries. (1) What is the distribution of X_2 ? (2) What is the probability that the mean distance it lands from me will be between 10-20 feet?
3. Let σ^2 denote the $Var[X_2]$, and let $X_3 = \frac{1}{\sigma^2}$ (called the **precision**). Suppose that the mean precision is 5 and the variance of the precision is 10. (1) What is the distribution family of X_3 ? Why does this make sense? (2) What is the probability that the precision is between 2-3?
4. Suppose that the average lifetime of any given lightbulb in Duncan Hall is 300 hours. Let X_4 be the lifetime of a lightbulb in Duncan Hall. (1) What is the distribution of X_4 ? (2) What is one unique property of this distribution, and what discrete distribution shares this property? (3) What is the variance of these lifetimes? (4) Suppose we define a “dud” lightbulb as lightbulbs that expire before even 1 hour of use. What is the probability that a given lightbulb is a dud?

2 Normal distributions.

A professor in a large statistics class has a grading policy such that only the 15% of the students with highest scores will receive the grade A. The mean score for this class is 72 with a standard deviation of 6. Assuming that all the grades for this class follow a normal probability distribution:

1. What is the minimum score that a student in this class has to get to receive an A grade?
2. What is the probability of receiving an A grade?
3. The professor lets the top 50% of students pass. What is the probability of passing?

4. If the students in the 60th-85th percentiles receive B grades, what is the probability of receiving a B grade?

3 Transformations of PDFs.

Find the distribution of the following random variables.

1. Let $X \sim \mathcal{N}(0, 1)$. Find the distribution of $Y = X^2$, using the transformation method.
2. Let $X \sim \text{gamma}(n, \beta)$. Find the distribution of $Y = 1/X$, using the transformation method.
3. **Probability integral transformation.** Let X have continuous CDF $F_X(x)$ and define the random variable $Y = F_X(X)$. Find the distribution of Y .