

STAT310  
Practice Problems  
Week 7

March 26, 2012

## 1 Bivariate PDFs.

Which of the following functions is a bivariate PDF?

1.  $f(x, y) = 2x$ ,  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ .
2.  $f(x, y) = x + y$ ,  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ .
3.  $f(x, y) = x^3 + \frac{y}{2}$ ,  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ .

## 2 Independent random variables.

For each of the following, determine whether the given PMF/PDF is the PMF/PDF of two independent random variables:

1. The joint PMF of  $(X, Y)$  given by

$$f(10, 1) = f(20, 1) = f(20, 2) = \frac{1}{10}$$

$$f(10, 2) = f(10, 3) = \frac{1}{5}$$

$$f(20, 3) = \frac{3}{10}.$$

2.  $f(x, y) = \frac{1}{384}x^2y^4e^{-y-(x/2)}$ ,  $x > 0, y > 0$ .
3.  $f(x, y) = c_1(xy + xy^3)$ ,  $c_1 \in \mathbb{R}, x \in [0, 1], y \in [0, 1]$ .
4.  $f(x, y) = c_2(x^2e^{y+x})$ ,  $c_1 \in \mathbb{R}, x \in [0, 1], y \in [0, 1]$ .

## 3 Conditional distributions.

1. Suppose  $(X, Y)$  has the joint PDF  $f(x, y) = e^{-y}$ ,  $0 < x < y < \infty$ . Find the marginal distribution of  $X$ .
2. Using the same joint PDF as in (1), find the conditional distribution of  $Y$  given  $X = x$ .
3. Suppose  $(X, Y)$  has the conditional PMF  $f(x|y) = \frac{x+y}{3y+6}$ ,  $x = 1, 2, 3$ ,  $y = 1, 2$ , and  $f_Y(y) = \frac{3y+6}{21}$ ,  $y = 1, 2$ . Find the joint PMF of  $(X, Y)$ .