STAT310
Practice Problems
Week 9

March 26, 2012

1 Limiting distributions.

1. Suppose $X_n \sim$N$(\mu_n = \frac{n}{n+1}, \sigma^2)$. Find the limiting distribution of X_n as $n \to \infty$.
2. Suppose $Y_n \sim$Gamma$(1 + e^{-n}, \beta)$. Find the limiting distribution of Y_n as $n \to \infty$.
3. Suppose $Z_n \sim$N$(\mu_n = 1, \sigma_n^2 = \sigma^2/n)$. Find the limiting distribution of Z_n as $n \to \infty$.

2 Law of large numbers and Chebyshev's inequality.

How large should n be so that we can be 90% certain that the sample mean, $\bar{X}_n$, of a series of iid random variables with mean μ and variance σ^2 will not deviate from the mean by more than $\frac{\sigma}{2}$?

3 Central limit theorem (CLT).

1. Suppose $X_1, ..., X_n$ are a random sample from a negative binomial$(r = 10, p = \frac{1}{2})$ distribution. Find $P(\bar{X} \leq 11)$ using the CLT.
2. Suppose $\bar{X}$ i the mean of 100 observations from a population with mean μ ad variance $\sigma^2 = 9$. Find limits between which $\bar{X} - \mu$ will lie with probability at least 0.90, using the CLT.

4 Moment generating functions.

1. Suppose $X_i \sim$iid Bernoulli(p), $i = 1, ..., n$, and put $Y = \sum_{i=1}^{n} X_i$. Find the MGF of Y.
2. Suppose $X_i \sim$ Unif$(0, a_i)$, independent, and put $Y = \sum_{i=1}^{n} X_i$. Find the MGF of Y. Does this correspond to a known distribution?