Stat310: Test 1

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
You have **two hours** and you may use one double-sided page of notes, and wolframalpha if you have to. You may not use your text book, or communicate anything about the contents of this test to anyone else. Please pledge on the last page.

Name: _____

1. On average, I call on three people at random in each class, and over the course of the semester we'll have 28 classes in total. There are 140 students in stat310.

   (a) What's the probability that I call on you two or more times during the semester? (4)

(b) What's the probability that **someone** in the class gets called on more than once in a semester. (Hint: think about the complement) (2)

(c) Should your answer to part (b) be big (close to 1) or small (close to 0)? Why? (2)

2. A helpful mathematical fact: $\sum_{i=0}^{\infty} x^i/i! = e^x$

(a) Verify that the pmf of the Poisson distribution, $f(x) = e^{-\lambda}\lambda^x/x!$ is a true pmf. (3)

(b) Complete the blanks to find the mean of the poisson distribution. (7)

$X \sim \text{Poisson}(\lambda)$

$E(X) =$ _____ by definition of $E(X)$

$=$ _____ by definition of X

$= \sum_{x=1}^{\infty} x \frac{e^{-\lambda} \lambda^x}{x!}$ _____

$= e^{-\lambda} \lambda$ _____ moving out constants

$=$ _____ $y = x - 1$

$=$ _____ _____

$= \lambda \square$

3. On average it takes me 10 minutes to drive to Rice from home.

   (a) Give three characteristics that the distribution of my driving times might reasonably possess. (3)

Assume that my driving times, D, has a exponential distribution. (This is not a very appropriate distribution, but it makes the rest of the question possible)

   (b) If I leave home at 12:45pm, what are my chances of making it to class by 1? (2)

   (c) What time do I need to leave home to have a 99% chance of making it to class on time? (2)

(d) Instead of looking at the distribution of time, I could look at the distribution of my velocity. It's 3.6 miles from my home to Rice. What's the distribution of my speed in miles per hour? Work it out by transforming the distribution of time. (5)

4. Compute the following three probabilities.

(a) Events A and B are a partition. $P(B) = 0.6$, $P(A \cap C) = 0.1$ and $P(B \cap C) = 0.2$. What is $P(A \cup C)$? (4)

(b) Let the random variable X have pmf $f_X(x) = \frac{6}{\pi^2 x^2}, x = 1, 2, \ldots$. What is $P(1 \leq X \leq 3)$? (3)

(c) Let the random variable Y have pdf $f_Y(y) = \frac{1}{y^2}$. What is $P(1 \leq Y \leq 3)$? (3)

Pledge (including time started and finished):

Question	Points	Score
1	8	
2	10	
3	12	
4	10	
Total:	40	