Stat310: Test 2

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
You have **two hours** and you may use one double-sided page of notes, and wolframalpha if you have to. You may not use your text book, or communicate anything about the contents of this test to anyone else. Please pledge on the last page.

Name: ______________________________

1. Let $X_1 \sim \text{Exp}(\theta)$, $X_2 \sim \text{Exp}(\theta)$ and X_1 and X_2 are independent. (Hint: $\Gamma(2) = 1$)
   (a) Using bivariate change of variables, find the pdf of $S_2 = X_1 + X_2$. What named distribution does this pdf represent? (7)

(b) Let $X_3 \sim \text{Exp}(\theta)$ and be mutually independent with X_1 and X_2. What is the distribution of $S_3 = X_1 + X_2 + X_3$? Use mgfs to find out. (3)

2. Let $H_i \sim \text{Normal}(\mu = 80, \sigma^2 = 25)$ be an iid sequence that represents your homework scores (as a percentage of 100).

   (a) Give three reasons why your homeworks are not independent. For each reason, characterise its effect on the correlation between scores. (3)

(b) Is the normal distribution an appropriate approximation to your homework scores? Why/why not? (2)

(c) Ignoring the qualms above, let $T_n = \sum_{i=1}^{n} H_i$ be your total homework score after n homeworks. What is the distribution of T_n? Justify mathematically. (3)

(d) Let $\bar{H}_n = T_n/n$, your average homework score. What assumptions can you relax so that $\bar{H}_n$ is approximately normally distributed? Why? (2)

3. Let $f(x, y) = c(x^2 + y^2)$, $x \in [0, 1]$, $y \in [0, 1]$. This pdf might represent your scores on the first and second homework as proportions between 0 and 1, no longer assuming independence.

   (a) What value of c makes f a valid bivariate pdf? What other properties do you need to check? (4)

(b) What is $P(X > 0.5 \cap Y > 0.5)$, the probability you get over 50% on both homeworks? (3)

(c) What is $E(\frac{X+Y}{2})$, the average grade on homeworks one and two? (3)

(d) Outline a strategy that would enable you to find $P(\frac{X+Y}{2} > 0.5)$. (2)

4. (a) Let $B_n \sim \text{Binomial}(n = n, p = \lambda/n)$, i.e. a binomial distribution where the number of successes is constant, but the size grows. Complete the blanks to find $\lim_{n\to\infty} B_n$. A helpful mathematical fact is that $\lim_{n\to\infty}(1+\frac{x}{n})^n = e^x$ (8)

$P(B_n = x) =$ __________ pmf of binomial

$$= \frac{n!}{(n-x)!\,x!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$ __________

$$= \underbrace{\frac{n!}{n^x(n-\cancel{x})!}}_{1}\underbrace{\left(\_\_\_\right)}_{2}\underbrace{\left(1-\frac{\lambda}{n}\right)^n}_{3}\underbrace{\left(1-\frac{\lambda}{n}\right)^{-x}}_{4}$$

$$\lim_{n\to\infty} 1 = \lim_{n\to\infty}\frac{n(n-1)(n-2)\cdots(n-\cancel{x}+1)}{n^{\cancel{x}}} = \lim_{n\to\infty}\left(\_\right)\left(\_\right)\cdots\left(\_\right)$$

$= $ ________

$\lim_{n\to\infty} 2 =$ ________

$\lim_{n\to\infty} 3 =$ ________ ________

$\lim_{n\to\infty} 4 =$ ________

$\Rightarrow \lim_{n\to\infty} P(B_n = x) =$ __________

$\Rightarrow \lim_{n\to\infty} B \sim$ Poisson(____)

Pledge (including time started and finished):

Question	Points	Score
1	10	
2	10	
3	12	
4	8	
Total:	40	