Stat310: Test 1

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
You have **80 minutes** and you may use one double-sided page of notes. You may not use your text book or a computer (calculators are fine). Please pledge on the last page.

Name: _____

1. I call my bank and the recording tells me there are 4 people ahead of me. I know on average that a call takes 3 minutes.
   (a) How long do I expect to wait? Why? (2)

   (b) What is a plausible distribution for the amount time a single call takes? Justify your choice. (3)

   (c) If I had a way of generating random numbers from a uniform distribution, how could I use them to simulate from distribution in b? (2)

   (d) Let N be the number of calls the bank receives in an hour. What's a plausible distribution for N? (3)

2. The following probability questions require you to explore some interesting findings about birthdays.

   (a) If there are 5 people in a room, what is the probability that any two of them share a birthday? (Hint: start by figuring out the probability that none of them share a birthday) (3)

   (b) If there are n people in room, what is the probability that any two of them share a birthday? (2)

   (c) There are some interesting seasonal patterns in birth date: while unmarried mothers are equally likely to give birth in any season, married mothers are more likely to give birth in spring and less likely to give birth in winter. Assume that only 0.075 of married mothers give birth in winter and about 30% of babies are born to unmarried mothers. Given that a child was born in winter, what's the probability that their mother was married? (5)

3. I've decided to decorate my office by painting randomly sized dots on the wall. For each dot, I randomly choose the radius by drawing it from a exponential distribution with mean 5. Some selected values of the cdf are shown in the table below.

X	$P(X \leq x)$	X	$P(X \leq x)$
1	0.18	11	0.89
2	0.33	12	0.91
3	0.45	13	0.93
4	0.55	14	0.94
5	0.63	15	0.95
6	0.70	16	0.96
7	0.75	17	0.97
8	0.80	18	0.97
9	0.83	19	0.98
10	0.86	20	0.98

(a) What's the probability that the radius of a dot is between 10 and 20cm? (2)

(b) What's the approximate probability that the area of a dot lies between 100 and 300 cm^2? (2)

(c) What size area do I need to leave blank to guarantee that at least 95% of the dots will fit in? (2)

(d) Work out the pdf for A, the area of each dot (4)

4. Let $f(x) = \frac{1}{2b}e^{-|x|/b}$, $x \in (-\infty, \infty), b > 0$. (Hint: if you get stuck, try sketching a plot of $f(x)$)

(a) Verify that $f(x)$ is a pdf. (5)

(b) Without using calculus, argue that the mean is 0. (2)

(c) Complete the steps below to work out the mgf. (3)

$M_X(t) =$ _____ _____

$=$ _____ by definition of expectation

$= \int_{-\infty}^{\infty} \frac{e^{tx}}{2b} e^{-|x|/b} \, dx$ _____

$=$ _____ $+$ _____ to remove absolute value sign

$= \frac{1}{2b}\left[\frac{b}{1-bt} + \frac{b}{bt+1} \right]$ _____

$=$ _____ common denominator

$=$ _____ □

Pledge:

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	