Stat310: Final exam

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
You have **120 minutes** and you may use three double-sided page of notes. You may not use your text book or a computer (calculators are fine). Please pledge on the last page, and note the times you started and ended.

Name: ______________________________

1. X is a random variable with $E(X) = \mu$ and $Var(X) = \sigma^2$. Let $Y = aX + b$. (5)
   What is cor(XY) if $a > 0$? If $a < 0$?

2. (a) Using the axioms of probability, show that $P(A \cup B) \leq P(A) + P(B)$. (3)

(b) When does $P(A \cup B) = P(A) + P(B)$? (1)

(c) If A and B are exhaustive, $P(A) = 0.4$, $P(B) = 0.7$, what is $P(A \cap B)$? (1)

3. (a) Using the definition of expectation, show that for any symmetric pdf (e.g. $f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$) all odd moments (e.g. $E(X)$, $E(X^3)$) are 0. (3)

(b) Show how to use the mgf to find out the value of the nth moment. (2)

4. The prostate specific antigen (PSA) test is used to detect prostate cancer. A "positive" test is consistent with cancer, and a "negative" with being disease free. The test has a sensitivity (the probability of being positive if the person has cancer) of 86%, and a specificity (the probably of being negative if the person doesnt have cancer) of 33%. If the overall rate of prostate cancer is 5%, what is the probability you have cancer if you have a positive PSA test? Is this a good test? (4)

5. Construct a 95% confidence interval for the sample mean if $\bar{x} = 100$, $s^2 = 20$, and $n = 100$. (Hint: $P(Z < 0.025) = 1.96$) (4)

6. (a) Let $X \sim \text{Normal}(\mu, \sigma^2)$. Using change of variables, find the pdf of $Z = \frac{X-\mu}{\sigma}$. (5)

(b) What is the distribution of Z? (1)

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Rayleigh distribution. A Rayleigh distribution has one parameter, $\sigma > 0$, mean $\sigma\sqrt{\frac{\pi}{2}}$ and pdf $f(x_i) = \frac{x_i}{\sigma^2}\exp(-\frac{x_i^2}{2\sigma^2})$

   (a) Find a maximum likelihood estimator of σ (You don't need to verify it's a maximum). (5)

   (b) Find a method of moments estimator for σ. (1)

8. Let $X_i \sim \text{Normal}(\mu_x, \sigma_x^2)$, $i = 1, \ldots, n$ and $Y_j \sim \text{Normal}(\mu_y, \sigma_y^2)$, $j = 1, \ldots, m$, and σ_x^2 and σ_y^2 are known constants (e.g. $\sigma_x^2 = 20$, $\sigma_y^2 = 15$)

   (a) Derive the distribution of $\bar{X} - \bar{Y}$. (This is a little more general than what we derived in class) (4)

   (b) Find a pivot for $D = \bar{X} - \bar{Y}$, and give its distribution. (1)

Pledge:

Time started: Time ended:

Question	Points	Score
1	5	
2	5	
3	5	
4	4	
5	4	
6	6	
7	6	
8	5	
Total:	40	