

## Stat310: Test 1

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You have **two hours** and you may use one double-sided page of notes. You may not use your text book or a computer (calculators are fine), or communicate anything about the contents of this test to anyone else. Please pledge on the last page.

Name: \_\_\_\_\_

1. On average, Ron Artest of the Houston rockets makes an attempt at a shot about every 90 seconds.
  - (a) What distribution might you use to model the waiting time for his next shot? (Make sure to include the parameter values) (2)
  
  
  
  
  
  
  
  
  
  
  - (b) What's the probability you wait for more than 5 minutes for his next attempt? (2)
  
  
  
  
  
  
  
  
  
  
  - (c) Instead of modelling the waiting time between attempts, I could model the **number** of attempts in a fixed amount of time. Let  $S$  be the number of shots he attempts in a five minute period. Assume that this is a Poisson distribution. What is the value of  $\lambda$ ? (1)
  
  
  
  
  
  
  
  
  
  
  - (d) What's the probability that he attempts zero shots in the next 5 minutes? (2)

(e) Compare your answers from b and d. Why are they different or the same? (1)

(f) List two questionable assumptions that I made when I assumed that this variable had a Poisson distribution. (2)

2. In last month, I received 997 spam emails and 2409 real emails. 11% of the spam messages contained the word bank, but only 1.5% of non-spam email contained it.

(a) Convert the word problem to a mathematical problem by setting up the events, and known probabilities, that you will use to answer the next questions. (1)

(b) What proportion of my email contains the word “bank”? (3)

(c) Given that an email contains “bank” what is the probability it’s spam? (3)

(d) If it doesn’t contain “bank”, what’s the probability it’s not spam? (3)

3. (a) Show that  $P(A|B) + P(\bar{A}|B) = 1$ . Explain your working. (5)

(b) Does  $P(A|B) + P(A|\bar{B}) = 1$ ? Why/why not? (3)

(c) If  $P(A) = .5$ ,  $P(B) = 0.5$  and  $P(A \cap B) = 0.25$  what does that imply about A and B? (1)

(d) Using the values above, what is  $P(\overline{A \cup B})$ ? (1)

4. (a) For the range  $x \in [0, 2]$ , which of the following functions is a valid pdf. If it's not a valid pdf, indicate why. (5)

(a)  $f_1(x) = c_1x^2$

(b)  $f_2(x) = c_2/x$

(c)  $f_3(x) = c_3(1 - x)$

(d) Fill in the blanks below to find the MGF for the exponential distribution.

(5)

$$M_X(t) = \underline{\hspace{10em}}$$

by definition of  
mgf

$$= \underline{\hspace{10em}}$$

by definition of  $E$   
and the pdf of the exponential

$$= \frac{1}{\theta} \int_0^{\infty} e^{x(t - 1/\theta)} dx$$

$$\underline{\hspace{10em}}$$

$$= \underline{\hspace{10em}}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$= \left[ \frac{e^{x(t - 1/\theta)}}{\theta t - 1} \right]_0^{\infty}$$

$$\underline{\hspace{10em}}$$

$$= \underline{\hspace{10em}} - \underline{\hspace{10em}}$$

definite integral

$$\lim e^{x^a} = 0 \text{ iff } a < 0 \Rightarrow 1/\theta > t$$

$$= \underline{\hspace{10em}} = \underline{\hspace{10em}}$$

□ if  $1/\theta > t$

Pledge (including time started and finished):

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	