Stat310: Test 2

Name: ______________________________

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

You have **two hours** and you may use one double-sided page of notes. You may not use your text book or a computer (calculators are fine), or communicate anything about the contents of this test to anyone else.

Pledge (including time started and finished):

1. Let X_1, X_2, ..., X_n be a sequence of independent random variables where $X_i \sim Exp(\theta_i)$

   (a) What is the mgf of $S_n = \sum_{i=0}^{n} X_i$? (2)

   (b) From here on assume that $\theta_i = \theta$ so the sequence becomes iid, and $M_{S_n}(t) = (1 - \theta t)^{-n}$. What is the distribution (name and parameters) of S_n? (2)

   (c) Let $\bar{X}_n = \frac{S_n}{n}$. What is the distribution (name and parameters) of $\bar{X}_n$? (2)

   (d) What does the LLN imply will happen to the variance of $\bar{X}_n$ as $n \to \infty$? What happens to the variance for this specific distribution of $\bar{X}_n$? (2)

   (e) How would you transform $\bar{X}_n$ to get Z_n so that the CLT applies? Can you write down the mgf of Z_n? (2)

2. Let $A \sim Normal(0, 1)$, and $B = c + dA$, where $-\infty < c < \infty$ and $d > 0$ are real numbers.

   (a) Find the distribution of B using the change of variables technique. (5)

   (b) Find the distribution of B using the mgf. (2)

(c) When considering a transformation (univariate or bivariate), give three factors to consider when selecting a technique to find the distribution of the transformed random variable. (3)

3. I'm interested in the weight of Rice squirrels. To collect some data, I have been walking around the inner loop and then whenever I see a squirrel, I race after it, wrestle it to the ground and then weight it with my pocket scales. Let $R_1, R_2, \ldots, R_n$ the be the random variables the represent the weights of squirrels captured using this process.

   (a) Would you be comfortable describing this sequence as iid? Why/why not? (2)

   (b) For the rest of the question, assume that the sequence is iid. What can you say about the distribution of W_i? What can you say about the distribution of $\sqrt(n)\bar{W}_n = \sqrt(n)\sum_{i=0}^{n} W_i/n$? (4)

(c) Estimate that probability that $\bar{X}_n$ is more than two standard deviations away from the population mean $\mu = E(X_i)$. Why is your estimate only approximate? (Hint: if $Var(X_i) = \sigma^2$, what is $Var(\bar{X}_n)$?) (3)

(d) What other technique could you have used to answer the part (c)? What's the advantage of the technique you chose? (1)

4. Imagine that I also measure the height of each squirrel, and I find that joint distribution of height in cm (H) and weight in g (W) is bivariate normal.

   (a) Approximately what value do you think ρ will be? Why? (2)

   (b) If ρ was zero, what would that imply about W and H? (1)

Assume that the parameters of the bivariate normal distribution are $mu_w = 700$, $\sigma_w^2 = 400$, $\mu_h = 20$, $\sigma_h^2 = 9$ and $\rho = 0.5$.

(c) Compute $Cov(W, H)$. (2)

(d) How would you prove that the marginal distributions are $W \sim Normal(\mu_w, \sigma_w^2)$ and $H\ Normal(\mu_h, \sigma_h^2)$? (Just outline the steps) (2)

(e) What's the probability that a randomly selected squirrel weighs more than 740g? (2)

(f) How would you compute the probability that a randomly selected squirrel weighs more than 740g **or** is taller than 26cm? (1)

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	