Stat310: Final

Name: ____

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Do not answer questions on a separate sheet of paper.

You have **three hours** and you may use three double-sided page of notes. You may not use your text book or a computer (calculators are fine), or communicate anything about the contents of this test to anyone else.

Due 5pm May 4.

Pledge (including time started and finished):

1. (a) Draw a venn diagram that illustrates the term mutually exclusive. (1)

   (b) Draw a venn diagram that illustrates the term partition. (1)

   (c) Show that it's hard to illustrate independence in a Venn diagram by creating two events X and Y that are independent, and two events A and B that are not independent, and both sets of events have the same Venn diagram. (2)

2. Verify that for an event Z with $P(Z) > 0$, the conditional probability function $P(A|Z)$ is a valid probability function.

   (a) Show that $P(A|Z) \geq 0$ for all $A \subset S$. (1)

   (b) Show that $P(S|Z) = 1$ (1)

   (c) Show that $P(A \cup B|Z) = P(A|Z) + P(B|Z)$ if $A \cap B = \emptyset$ (2)

3. (a) Using the change of variables technique (or otherwise), find the pdf of X if $X = aZ + b$ and $Z \sim \text{Normal}(0, 1)$. (4)

(b) What is the distribution of X? (1)

4. The waiting time, in minutes, between Lola's barks can be modelled as $W \sim \text{Exponential}(\theta = 10)$.

   (a) What's the probability that Lola waits less that 10 minutes (the expected value) before barking next? (1)

   (b) Given that Lola hasn't barked in the last hour, what's the probability that she barks in the next 5 minutes? (3)

   (c) How many times do you expect Lola to bark in one day? (1)

   (d) What's the distribution of the number of times Lola barks in a day? (1)

5. Let X and Y be independent random variables with distributions $X \sim$ Binomial(n, p) and $Y \sim$ Binomial(m, q). Let $Z = X + Y$

   (a) What's the mgf of Z? (Hint: the mgf of X is $(1 - p + pe^t)^n$) (1)

   (b) What's the distribution of Z if $p = q$? (2)

   (c) What's the distribution of Z if $m = n$? (1)

   (d) How would you find the distribution of Z if X and Y weren't independent? (1)

6. Given $X_i \overset{iid}{\sim}$ Bernoulli(p), show that the maximum likelihood estimator for p is $\bar{X}_n = \sum_{i=1}^n X_i/n$. (You don't need to verify that the extremum you find is a maximum, but if you do you'll get a bonus point.) (5)

7. Let the event that a stat310 student is in class when I call their name in a your turn be modelled as $C_i \overset{iid}{\sim} Bernoulli(p)$, where p is the probability that a student is in class.

   (a) Over the course of the year, I called out 64 names in your turns and 49 people responded. What is the maximum likelihood estimate of p? (1)

   (b) We know that $\hat{p} \dot{\sim} \text{Normal}(p, p(1-p)/n)$. Use this to construct a 95% confidence interval for p. (Hint: $\Phi(-1.96) = 0.025$) (4)

   (c) We could also try to construct a confidence interval using our knowledge that $n\bar{X} \sim Binomial(n, p)$. Why won't this work? (1)

8. I collected 15 numbers from an experiment that can be well modelled with a Normal distribution with mean 0 and unknown variance. The mean of the 15 numbers was 0.035 and the variance (S^2) was 0.71. Construct a hypothesis test to see if the numbers came from a distribution with variance 1, or if the real variance was bigger than that. (5)

CDF of $\chi^2(14)$

x	$P(X \leq x)$
8	0.11
9	0.17
10	0.24
11	0.31
12	0.39
13	0.47
14	0.55
15	0.62

Question	Points	Score
1	4	
2	4	
3	5	
4	6	
5	5	
6	5	
7	6	
8	5	
Total:	40	