Stat310: Final

Name: _____

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

You have **three hours** and you may use one double-sided page of notes, and wolframalpha if you have to. You may not use your text book, or communicate anything about the contents of this test to anyone else. Please pledge below.

A standard normal table is included on the last page of the test in case you need it.

Due 5pm May 2.

Pledge (including time started and finished):

1. When checking the weather forecast on my phone, I saw that there was a 50% probability of rain for each of the next six hours.

   (a) If each hour was independent, what would be the probability that I need to bring my umbrella? (i.e. what's the probability it rains at least once during the next six hours?) (2)

   (b) Do you think each hour really is independent? Why/why not? What impact do you think that would have on the probability of me needing an umbrella? (2)

2. In stat310, on average on 98 out of 128 students turn up for class. If you're one of the students who do turn up for class, you'll know that when I pick on someone at random, I often have a few misses before I find someone who's actually in class. Let C be number of names I call each time.

   (a) What's the distribution of C? (2)

   (b) What's the probability I only have to call one or two names? (2)

   (c) What's the longest sequence of names I'd expect to call over the course of the semester? (There are 28 classes, and you can assume I call on 3 people each class.) (1)

3. (a) Find the mgf of D if $D = A + B$, $A \sim$ Normal(μ_x, σ_x^2), $B \sim$ Normal(μ_y, σ_y^2) and A and B are independent. (4)

(b) What is the distribution of D? (1)

4. If $X_1, X_2, \cdots, X_n$ are an iid sequence of negative binomial random variables with known n and unknown p, find the maximum likelihood estimator of p. (The pmf of the negative binomial is $\binom{x+r-1}{x}(1-p)^r p^x$). (You don't need to prove that it's a maximum, unless you want a bonus point). (6)

5. Last season, on average, Ron Artest attempted 14.9 shots per game, made 58% of them. Let S be the number of shots he attempted, and M the number of shots he made.

   (a) If the number of shots he attempts has a Poisson distribution, and given that he attempts s shots, M has a binomial distribution with $n = s$, what's the joint distribution of M and S? (2)

   (b) How would you find the distribution of M? (1)

   (c) Instead of thinking about the number of shots he makes, we could think about the time we have to wait between shots. Given that a game of basketball lasts 48 minutes, what is the distribution of W, the amount of time you have to wait between **successful** shots? (2)

6. Between 2000 and 2012, Will Rice won 2, 1, 1, 1, 2, 0, 1, 1, 2, 3, 2, 2, and 2 out of the three beer bike titles. Test the hypothesis that their probability of winning a single title is greater than 0.5. You can assume that an estimator of p is $\bar{X}_n/3$. (5)

7. (a) Compare and contrast the LLN to the CLT. (2)

(b) Derive the variance and expectation of $\bar{X}_n$ when the X_i's are iid. (2)

(c) Transform $\bar{X}_n$ so that has (approximately) a standard normal distribution. What additional condition on the X_is makes $\bar{X}_n$ exactly normally distributed? (2)

8. (a) Why do we fail to reject the null hypothesis instead of accepting the alternative? (1)

(b) What two basic conditions do we check to validate that a function is a probability function, pdf, pmf or bivariate pdf? (1)

(c) How are the Gamma and Poisson distributions related? (1)

(d) How do you find the cdf from the pdf? How do you find the pdf from the cdf? (1)

Probability Content from -oo to Z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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Question	Points	Score
1	4	
2	5	
3	5	
4	6	
5	5	
6	5	
7	6	
8	4	
Total:	40	